Physikalisches Institut
Universität Bonn
Theoretische Physik

Exercises 4
10 May 2010
SS 2010

Exercises on Theoretical Particle Physics II

Priv.Doz.Dr. S.Förste

Due 17/05/2010

4.1 Vectorsuperfields and gauge transformations (*10 credits*)

Global $\mathcal{N}=1$ supersymmetry allows one more supermultiplet, the **vectormultiplet**, which contains a supersymmetric version of a gauge theory. It consists of the usual spin one gauge boson V_μ as well as its spin one half superpartner λ called the **gaugino**. There also exists a superfield formulation of the vectormultiplet completely analogous to the chiral superfield describing the chiral multiplet $(\varphi,\,\psi)$. The appropriate superfield V is the **vectorsuperfield** defined by $V=V^\dagger$ with the expansion

$$\begin{aligned}
V(x,\theta,\bar\theta) \;=\;& C(x)+i\theta\chi(x)-i\bar\theta\bar\chi(x)-\theta\sigma^\mu\bar\theta V_\mu(x)\\
&+\frac{1}{2}i\theta\theta\left[M(x)+iN(x)\right]-\frac{1}{2}i\bar\theta\bar\theta\left[M(x)-iN(x)\right]\\
&+i\theta\theta\bar\theta\left[\bar\lambda(x)+\frac{i}{2}\bar\sigma^\mu\partial_\mu\chi(x)\right]-i\bar\theta\bar\theta\theta\left[\lambda(x)+\frac{i}{2}\sigma^\mu\partial_\mu\bar\chi(x)\right]\\
&+\frac{1}{2}\theta\theta\bar\theta\bar\theta\left[D(x)+\frac{1}{2}\partial_\mu\partial^\mu C(x)\right]. \qquad (1)
\end{aligned}$$

(a) Check that (1) is indeed a vectorsuperfield. (*3 credits*)

(b) Compare the expansion of V to the one of the vectorsuperfield defined by $\Lambda+\Lambda^\dagger$ where $\Lambda_L(x,\theta)=\Lambda(x)+\sqrt{2}\theta\psi_\Lambda(x)+\theta\theta F_\Lambda(x)$ is a left-chiral superfield, here given in the left-chiral representation. What is the interpretation of the transformation

$$V\mapsto V+\Lambda+\Lambda^\dagger\ ? \qquad (2)$$

Write the transformation law for all the component fields of V. (*4 credits*)

Hint: Work in the left-chiral representation by shifting the argument x^μ of $\Lambda^\dagger$. How does V_μ transform?

(c) Give the appropriate Λ in (2) to transform V into the **Wess-Zumino gauge** V_{WZ}, i.e. to obtain $C(x)=\chi(x)=M(x)=N(x)=0$. What is the highest non-vanishing power of V_{WZ}? Calculate V_{WZ}, V_{WZ}^2 as well as V_{WZ}^3. (*3 credits*)

4.2 Supersymmetric U(1) gauge theory. (*10 credits*)

To construct an action exhibiting the gauge symmetry (2) we have to find an adequate gauge invariant quantity. This will be the building block of any gauge invariant action. Exploiting the gauge invariance of the gaugino λ w.r.t. (2) in WZ-gauge we define the supersymmetric field strength of V by

$$W_\alpha = -\tfrac{1}{4}\bar{D}\bar{D}D_\alpha V, \qquad \bar{W}_{\dot{\alpha}} = -\tfrac{1}{4}DD\bar{D}_{\dot{\alpha}}V. \tag{3}$$

Note that the lowest component (in θ, $\bar{\theta}$) of W_α is the gauge invariant gaugino λ_α.

(a) Show that W_α in (3) defines a gauge invariant, left-chiral superfield! How do W_α, $\bar{W}_{\dot{\alpha}}$ transform under Lorentz transformations? Expand W_α in its component fields. (*5 credits*)

Hint: Translate V_{WZ}, D_α and $\bar{D}_{\dot{\alpha}}$ into the left-chiral representation. Finally use $\sigma^\mu\bar{\sigma}^\nu - \eta^{\mu\nu} = -2i\sigma^{\mu\nu}$.

(b) The simplest SUSY, gauge as well as Lorentz-invariant action for a vectorsuperfield reads

$$S_{\mathrm{U}(1)} = \int d^4x d\theta^2 W^\alpha W_\alpha. \tag{4}$$

Why is this SUSY-invariant? Determine its component expression

$$\mathcal{L}_{\mathrm{U}(1)} = -\tfrac{1}{2}F^{\mu\nu}F_{\mu\nu} - 2i\lambda\sigma^\mu\partial_\mu\bar{\lambda} + D^2 + \tfrac{i}{4}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}, \tag{5}$$

with field strength $F_{\mu\nu}$. The last term is imaginary and cancels after adding the h.c. to (4). Note also the presence of the new auxiliary field D. (*5 credits*)

Hint: Use the identities $tr(\sigma^{\mu\nu}) = 0$ and $\sigma^{\mu\nu}\sigma^{\rho\sigma} = \frac{1}{4}(\eta^{\mu\rho}\eta^{\nu\sigma} - \eta^{\mu\sigma}\eta^{\nu\rho}) - \frac{i}{4}\epsilon^{\mu\nu\rho\sigma}$.