Physikalisches Institut
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Theoretische Physik

Exercises 10
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Exercises on Theoretical Particle Physics II

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10.0 Supergravity definitions (*0 credit*)

Lets us introduce some useful relations to solve the problems.
The Kähler potential $K(\Phi^i, \Phi^*_j)$ and the superpotential $W(\Phi^i)$ are combined to

$$G(\Phi^i, \Phi^*_j) = -\frac{K(\Phi^i, \Phi^*_j)}{M^2} - \log(\frac{|W(\Phi^i)|^2}{M^6}).$$

The F-term part of the scalar potential is given by:

$$V_{scalar}(\phi^i, \phi^*_j) = -e^G[3 + G_k(G^{-1})^k_l G^l]M^4,$$

where ϕ denotes the scalar component of Φ and the derivatives are defined as

$$G_i = \frac{\partial G}{\partial \phi^{*i}}, G^j = \frac{\partial G}{\partial \phi_j}, G^j_i = \frac{\partial^2 G}{\partial \phi^{*i}\partial \phi_j},$$

Furthermore, the F_i component of the chiral superfield Φ^i (for constant gauge kinetic function f_{AB}) reads:

$$F^i = W^i + \frac{1}{M^2}K^i W$$

10.1 The Polonyi Model (*11 credits*)

A **hidden sector** is defined as sector of the theory that is coupled to the matter or **observable sector** only by gravitational interactions. This hidden sector is usually used for supersymmetry breaking as we will investigate for the Polonyi model.
The hidden sector is defined by a single chiral superfield Φ that is a complete gauge singlet. For simplicity, assume a minimal Kähler potential $K(\Phi, \Phi^*) = \Phi^*\Phi$ and the following **Polonyi superpotential**

$$W(\Phi) = m^2(\Phi + \beta) \tag{1}$$

Fwd:with β a dimensionful parameter which we will use later to adjust the vacuum energy to zero.

(a) Determine the F-term of Φ. For which values of β is SUSY definitely broken? (*3 credits*)

(b) For which values of β is there a non-SUSY vacuum with zero energy? Calculate the VEV of ϕ at those vacua. (*4 credits*)
Hint: The solution should be $\langle\phi\rangle_\pm = \pm(\sqrt{3}-1)M$ *and* $\beta_\pm = \pm(2-\sqrt{3})M$.

(c) Calculate the gravitino mass $m_{3/2} = e^{-G/2}M$. Discuss its order of magnitude. Express it in terms of the SUSY breaking scale $M_{\text{SUSY}}^2 = \langle e^{-G/2}(G^{-1})_l^k G_k M\rangle$. (*2 credits*)

(d) Determine the scalar mass m_ϕ in terms of $m_{3/2}$ by expanding around $\langle\phi\rangle$. Finally, calculate the supertrace, including the gravitino.[1] Compare to F-term breaking in the globally supersymmetric case! (*3 credits*)

This situation changes dramatically in the case of more than one chiral multiplet. For N chiral multiplets the supertrace reads

$$\text{STr}M^2 = 2(N-1)m_{3/2}. \tag{2}$$

Thus, the scalar SUSY partners have to be more massive than the fermions in the multiplet what is required by the experimental data about the observed particle spectrum up to now. Furthermore, it is possible, due to Ex. 10.1 (c), to obtain a much smaller gravitino mass $m_{3/2}$ than the SUSY breaking scale M_S. Hence, a mass splitting between bosons and fermions of the right order of magnitude seems possible.

10.2 No Scale Model (*11 credits*)

Take $\mathcal{N} = 1$ supergravity with three chiral superfields S, T and C. The Kähler potential (with $M_P = 1$) is

$$K = -\log(S + S^*) - 3\log(T + T^* - CC^*) \tag{3}$$

The superpotential is

$$W = C^3 + a\exp(-\alpha S) + b, \tag{4}$$

where a and b are arbitrary complex numbers and $\alpha > 0$. These additional terms will enable us to
fix $\langle S\rangle$.

(a) Find the auxiliary fields for S, T and C and check that SUSY is broken. (*3 credits*)

(b) Calculate the scalar potential. (*2 credits*)

(c) What is the value of the vacuum energy? Are there at directions (where E_{vac} is independent of the vev of a field)? (*4 credits*)

(d) What is the gravitino mass? (*2 credits*)

[1]The fermion of the superfield Φ is eaten by the gravitino.