Exercises on Theoretical Particle Physics II

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Mock Exam, to discuss after 19/07/2010

1 Overview

Give a short answer to the following questions.

(a) Which superfield functions are required to completely determine an $\mathcal{N} = 1$ SUSY model? How do they enter the Lagrangian? Explain why those contributions are supersymmetric.

(b) Which building blocks are used to construct the MSSM, and how are the fields of the SM embedded? Why is it necesary to introduce an $R-$symmetry? Explain the concept.

2 Massive representations of Supersymmetry

In order to explore the particle content of SUSY theories we will use the *Wigner Method*. For that purpose one identifies for a given 4-momemtum q^μ a representation of a certain subgroup H of the SUSY group which leaves q^μ invariant. Then one constructs the representations of H on the $|q^\mu\rangle$ states. Let us consider the supersymmetry algebra with massive irreps., no central charges, and $\mathcal{N}$ supersymmetries.

(a) Take as the rest-frame momentum $q_s^\mu = (m, 0, 0, 0)$. Which are the generators of the subset H which leave q_s^μ invariant?

(b) Check that the supersymmetry algebra acting on the rest-frame states is:

$$\begin{aligned}
\{Q^{\alpha i}, \bar{Q}^{\dot{\beta} j}\} &= 2\delta^{\alpha\dot{\beta}}\delta^i_j m \qquad (1)\\
\{Q, Q\} = 0, &\qquad \{\bar{Q}, \bar{Q}\} = 0\\
[J_m, J_n] &= \epsilon_{mnr} J_r\\
\left[Q^{\alpha i}, J_m\right] &= i(\sigma_m)^\alpha_\beta Q^{\beta i}\\
\left[\bar{Q}^{\dot{\alpha} i}, J_m\right] &= i(\sigma_m)^{\dot{\alpha}}_{\dot{\beta}} \bar{Q}^{\dot{\beta} i}
\end{aligned}$$

(c) Consider now the Clifford vacuum $Q^i_\alpha|q_s^\mu\rangle = 0 \ \ \forall\ \alpha, i$. Construct the states by appliying the creation operators $a^{j\dagger}_\alpha = \bar{Q}^j_{\dot{\alpha}}/\sqrt{2m}$ on the vacuum. How many states do you have for a given $\mathcal{N}$? Which $a^{j\dagger}_\alpha$'s lower the J_3 component of the angular momemtum, and which raise it?

(d) For the case $\mathcal{N} = 1$ and a Clifford vacuum with spins $j = 0, \frac{1}{2}$, construct the irreps. and specifiy their spin.

3 Superfields

We start from the general expansion of a superfield F and the differential operator $\bar{D}_{\dot{\alpha}}$:

$$\begin{aligned} F &= f(x) + \theta\phi(x) + \bar{\theta}\bar{\chi}(x) + \theta^2 m(x) + \bar{\theta}^2 n(x) + \theta\sigma^\mu\bar{\theta}V_\mu(x) + \theta^2\bar{\theta}\bar{\lambda} + \\ &+ \bar{\theta}^2\theta\psi(x) + \theta^2\bar{\theta}^2 d(x), \\ \bar{D}_{\dot{\alpha}} &= -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} - i\theta^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu. \end{aligned} \tag{2}$$

(a) Impose the left-chirality constraint $\bar{D}\Phi = 0$, with $\bar{D}$ in the non-chiral representation, to show that the component fields are not constrained by differential equations in x^μ.

(b) Choose the left-chiral representation for the covariant derivatives $y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta}$. Obtain the expresion for $\bar{D}_{\dot{\alpha}}$ in this representation.

(c) Deduce the general form of a left-chiral superfield in coordinates y^μ and x^μ.

4 The Wess-Zumino model

The Lagrangian of the Wess-Zumino model is given by:

$$\mathcal{L} = \int d^2\theta d^2\bar{\theta}\phi\phi^\dagger + \left[\int d^2\theta m\phi^2 + \lambda\phi^3 + \text{h.c.}\right]. \tag{3}$$

(a) Calculate the superpotential using the left-chiral representation. Argue that the shift which connects the regular and the left-chiral representations does not change the Lagrange density.

(b) Calculate the contributions from the Kähler potential. Use again the left-chiral representation.
Hint: Use $\phi\phi^\dagger = \phi_L(x,\theta)\exp[-2i\theta\sigma^\mu\bar{\theta}\partial_\mu][\phi_L(x,\theta)]^\dagger$

(c) Calculate the EOM for the F-field and use the result to eliminate F from the Lagrange density given in (3).

(d) Show that the scalar potential $V(\phi)$ is obtained from the superpotential via

$$V(\phi) = \left|\frac{\partial W(\varphi)}{\partial\varphi}\right|^2, \tag{4}$$

where $W(\varphi)$ means to take the superpotential $W(\phi)$ and set all components but φ to zero.

5 D-term SUSY Breaking

We consider the Lagrangian

$$\mathcal{L} = \int d^2\theta d^2\bar{\theta}[\Phi^\dagger e^{2qV}\Phi + 2\xi V] + \frac{1}{4}\int d^2\theta W^\alpha W_\alpha + \text{h.c.} \tag{5}$$

where $\Phi = (\varphi, \psi, F)$ is a chiral superfield, V a vector superfield (use WZ gauge), q is the $U(1)$ charge of Φ, and ξ is a real parameter (the Fayet-Iliopoulos parameter).

(a) Calculate $\mathcal{L}$ and the D-term equation of motion from (5). What is the scalar potential $V(\varphi)$?
Hint: Expand $e^{2qV} = 1 + 2qV + 2q^2V^2 + \ldots$ *and work in WZ gauge or use the result from the exercise sheets.*

Now we want to discuss the two possible cases (i) $q\xi < 0$ and (ii) $q\xi > 0$.

(b) Which symmetries are broken in the cases (i) and (ii)?

(c) In **case (i)**, we see from the shape of the scalar potential $V(\varphi)$ that its radial component gets massive while its angular component stays massless. Verify this via a computation.

(d) In **case (ii)**, all particles except φ stay massless. Show this via explicit calculation. Also calculate the mass of φ.

6 A no-scale supergravity model

Consider the Kähler potential $K = -\ln(S + S^*) - 3\ln(T + T^*)$, and the superpotential

$$W = -w + \kappa m_{SU}^3 \exp(-3S/|b_8|), \tag{6}$$

with w, k arbitrary real quantities, and $|b_8|$ a real positive number.

(a) Compute the scalar potential, and simplify it to get

$$V = \frac{(S + S^*)|F_S|^2}{(T + T^*)}. \tag{7}$$

(b) For which values of S and T is the potential minimized? What is the value of the potential at the minimum?

(c) Write down the gravitino mass in terms of the VEVs at the minimum.

(d) Is this a supersymmetric vacuum? Argue why / why not.

(e) Explain why this is called a *no scale* model.