Physikalisches Institut
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Theoretische Physik

Exercise 10
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SS 2011

Exercises on General Relativity and Cosmology

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Due 29 June 2011

Exercise 10.1: Einstein equation (*2 credits*)

(a) In a local region of spacetime, an observer finds that the Ricci scalar is nearly constant, $R \approx +1/a^2$. What can you say about the equations of state of the matter sourcing this curvature? If the region of spacetime is filled only with electromagnetic energy, what is R? (*1 credit*)

(b) Is it possible to have a solution of the Einstein equations, in which space is empty to the past of some surface of constant time $t = 0$, but in which there is a nonvanishing $T_{\mu\nu}$ to the future of this surface? (*1 credit*)

Exercise 10.2 Motion in Schwarzchild geometry (*18 credits*)

(a) A particle falls radially into a Schwarzschild metric.
(i) As measured by proper time at infinity, what is its inward coordinate velocity (dr/dt) at a (curvature-) radius r? (*3 credits*)
(ii)What is the locally-measured velocity relative to a stationary observer at the same radius? (*2 credits*)

(b) Derive the equations of motion (ie. equations relating t, r and proper-time τ) for a particle falling radially in the Schwarzschild geometry. Consider the three cases:
(i) particle released from rest at $r = R$ (*3 credits*)
(ii) particle released from rest at infinity (*3 credits*)
(iii) particle projected inward from infinity with velocity v_∞. (*2 credits*)

(c) Show that the trajectory of light rays in the Schwarzschild metric obeys:

$$\frac{d^2u}{d\phi^2} + u = 3u^2$$

where, $u = M/r$ and r is the Schwarzschild radial coordinate. Denote the minimum value of r along the trajectory by b, the *impact parameter*. In case of $(M/b) << 1$, what is the deflection of a photon as it passes a spherical gravitating body? Give a formula for the deflection angle to lowest nonvanishing order in (M/b). (*5 credits*)