Physikalisches Institut
Universität Bonn
Theoretische Physik

Exercise 3
12.05.2020
Summer term 2020

Theoretical Particle Astrophysics

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https://www.th.physik.uni-bonn.de/people/berbig/SS20-TAPP/

Homework
Due May 19th, 2020

H.3.1 Quickies *(4 points)*

a) State the definition of the critical density ρ_c. What happens, when the energy densities of matter, radiation and vacuum energy in the universe today add up to the critical density? (*2 points*)

b) State the definition of the particle horizon and the event horizon. For which kinds of cosmologies can one use the aforementioned horizons? (*2 points*)

H.3.2 Accelerated Expansion in a de-Sitter universe *(6 points)*

The de-Sitter cosmology is defined in terms of the following invariant line element

$$\mathrm{d}s^2 = \mathrm{d}t^2 - \exp\left(2H_{dS}t\right)\mathrm{d}\vec{x}^2 \quad \text{with} \quad H_{dS} > 0. \tag{1}$$

a) Show that the Riemann curvature tensor for this metric is given by

$$R_{\mu\nu\lambda\rho} = H_{dS}^2\left(g_{\mu\rho}g_{\nu\lambda} - g_{\mu\lambda}g_{\nu\rho}\right). \tag{2}$$

(*4 points*)

b) Use the above result to determine the Einstein field equations to show that this universe is filled with a positive vacuum energy. (*2 points*)

H.3.3 Age of the universe for some toy cosmologies *(10 points)*

In the following we will compute the age of the universe for two different scenarios.

a) Our first cosmology is spatially open and defined in terms of the following present day energy densities[1]

$$\Omega_{\text{mat.}} \neq 0, \quad \Omega_{\text{rad.}} = 0, \quad \Omega_{\text{curv.}} \neq 0, \quad \Omega_{\Lambda} = 0 \quad \text{and} \quad \Omega_{\text{mat.}} + \Omega_{\text{curv.}} = 1. \tag{3}$$

[1] This particular cosmology is excluded by measurements of the CMB.

Use the Friedmann equation to find ananalytical expression for the age of the universe today as a function of the energy densities and the present day Hubble rate H_0. Calculate its numerical value for $H_0 = h \times 100\,\mathrm{km\,Mpc^{-1}\,s^{-1}}$ with $h = 0.7$ and $\Omega_{\mathrm{mat.}} \approx 0.3$.
Hint: It is useful to define a dimensionless, time dependent scale factor $A(t)$ via the relation $a(t) = A(t) \cdot x$. Here the constant x has the dimension of length and can be chosen such that $a(t_0) = x$, which implies $A_0 \equiv A(t_0) = 1$. (*5 points*)

b) The next universe is spatially flat and has an energy budget given by

$$\Omega_{\mathrm{mat.}} \neq 0, \quad \Omega_{\mathrm{rad.}} = 0, \quad \Omega_{\mathrm{curv.}} = 0, \quad \Omega_\Lambda \neq 0 \quad \text{and} \quad \Omega_{\mathrm{mat.}} + \Omega_\Lambda = 1. \tag{4}$$

Assume that the present day energy density Ω_Λ comes from dark energy with an equation of state $P_\Lambda = \omega\, \rho_\Lambda$, where ω remains constant. Use the continuity equation

$$\dot{\rho_\Lambda} + 3\frac{\dot{a}}{a}\left(\rho_\Lambda + P_\Lambda\right) = 0 \tag{5}$$

to deduce how ρ_Λ depends on the scale factor and consequently how Ω_Λ appears in the Friedmann-equation. (*2 points*)

c) Use your findings to calculate the age of the universe for the previous scenario. Due to complicated nature of the resulting expression it suffices to give a numerical estimate for $\omega = -1.1$ and $\omega = -0.9$ by using $\Omega_{\mathrm{mat.}} = 0.27$ and $h = 0.7$. (*3 points*)