Theoretical Particle Astrophysics

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`https://www.th.physik.uni-bonn.de/people/berbig/SS20-TAPP/`

Homework

Due June 9th, 2020

H.5.1 Quickies *(2 points)*

a) Briefly explain why the temperature of about $0.3\,\mathrm{eV}$, at which the production of neutral hydrogen becomes thermodynamically favored, is less than one would naively expect from the binding energy of neutral hydrogen, which is $13.6\,\mathrm{eV}$. (*1 point*)

b) The baryon asymmetry of our universe can be quantified with the time independent ratio $\Delta_B = \frac{n_B - n_{\overline{B}}}{s}$, where n_B ($n_{\overline{B}}$) is the number density of baryons (anti-baryons) and s is the entropy density of the universe. It turns out that $\Delta_B \approx 0.14\,\eta_B$, where $\eta_B \approx 6.1 \times 10^{-10}$ is the baryon to photon ratio inferred from measurements of the CMB and nuclear abundances from BBN. What does the smallness of this number imply for the chemical potential of baryons μ_B at temperatures much larger than both μ_B and the baryon mass? (*1 point*)

H.5.2 Time of recombination from photon last scattering *(8 points)*

Previously we defined recombination as the moment in time, when the production of neutral hydrogen in the early universe became effective. From a cosmological point of view it is more interesting to know when the CMB photons interacted with matter for the last time. The corresponding temperature of last scattering is found to be $T_r = 0.26\,\mathrm{eV}$.

a) First assume that the expansion of the universe was only driven by non-relativistic matter with a present day energy density of $\Omega_{\text{mat.}} = 0.27$. How does the energy density of non-relativistic matter redshift with temperature? Show that the Hubble rate at last scattering is

$$H(T_r) = 4.45\,\mathrm{Mpc}^{-1} \tag{1}$$

and find the age of the universe at this time. (*3 points*)

b) Now consider a universe filled with both radiation and non-relativistic matter. How does the energy density of radiation redshift with temperature? Determine the Hubble rate as a function of temperature for present day energy densities $\Omega_{\text{mat.}} = 0.27$ and $\Omega_{\text{rad.}} = 10^{-4}$. Numerically integrate the Friedmann-equation to determine the age of the universe at photon last scattering. (*4 points*)

c) Why should you include both matter and radiation in the Friedmann equation when determining the age of the universe at last scattering? (*1 point*)
Hint: When does matter-radiation-equality take place?