Physikalisches Institut
Universität Bonn
Theoretische Physik

Exercise 6
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Theoretical Particle Astrophysics

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https://www.th.physik.uni-bonn.de/people/berbig/SS20-TAPP/

Homework
Due June 16th, 2020

H.6.1 Quickies *(2 points)*

a) Briefly explain the horizon problem associated with the horizon of photon last scattering in your own words. (*2 points*)

H.6.2 Decoupling of electron- and muon-neutrinos *(10 points)*

Consider a thought experiment, where you neglect the existence of the Z-boson as well as neutrino oscillations and loop processes. Furthermore focus on the electron- and muon-neutrinos.

a) Which Standard Model processes can keep the electron- and muon-neutrinos (as well as their antineutrinos) in thermal equilibrium with the plasma at temperatures above the muon-mass? Assume that the temperature is below the QCD phase transition and that all hadronic degrees of freedom in the plasma have already decayed away. (*2 points*)

b) At which temperatures do the processes keeping muon-neutrinos in thermal equilibrium become ineffective? What is the number of relativistic degrees of freedom in the plasma before and after this happens? (*3 points*)
Hint: Find a kinematic reason why the scattering reactions for muon-neutrinos are not in thermal equilibrium anymore.

c) The scattering rate for the processes connecting electron-neutrinos to the Standard Model plasma is $\Gamma = G_F^2 T^5$, where G_F is Fermi's constant. Estimate the Hubble rate as $H = \frac{T^2}{M_{\text{Pl}}}$ with the Planck mass given by $M_{\text{Pl}} = 1.22 \times 10^{28}\,\text{eV}$ and find the decoupling temperature of electron-neutrinos. (*1 point*)

d) At temperatures below the electron mass electrons and positrons annihilate into photons. State the number of relativistic degrees of freedom in the plasma before and after electron-positron-annihilation. Use the conservation of entropy in the comoving volume ($s\,a^3 \propto g_* T^3 a^3 = \text{const.}$) to find the temperature of the decoupled electron-neutrinos in terms of the present day photon temperature. (*2 points*)

e) Find the temperature of the decoupled muon-neutrinos in terms of the present day photon temperature. Which of the two relic neutrino flavors has the larger number density? (*2 points*)

H.6.3 Graviton Decoupling (*8 points*)

Speculative theories of quantum gravity postulate the existence of a spin 2 particle mediating the gravitational interaction called the graviton. Here we will estimate the relic abundance of such a graviton.

a) Based on dimensional analysis purely graviational interactions of the graviton can be parameterized by the following scattering rate

$$\Gamma = \frac{T^5}{M_{\text{Pl}}^4}. \tag{1}$$

Compare this to the Hubble rate $H = \frac{T^2}{M_{\text{Pl}}}$ to find the decoupling temperature of gravitons. (*1 point*)

b) The number of relativistic degrees of freedom needed for the computation of the **total** entropy density g_{*s} reads

$$g_{*s} = \sum_{\text{bosons}} g_i \left(\frac{T_i}{T}\right)^3 + \frac{7}{8} \sum_{\text{fermions}} g_i \left(\frac{T_i}{T}\right)^3. \tag{2}$$

If all particles are in thermal equilibrium ($T = T_i$ for each species) this reduces to the definition of g_* from the lecture. Show that the number of relativistic degrees of freedom at the graviton decoupling temperature is 106.75. At these temperatures all Standard Model particles would have been relativistic and in thermal equilibrium with each other. Neglect the contribution of the graviton to g_{*s}. (*4 points*)

c) Find g_{*s} for the universe today in which only the photon and three generations of decoupled neutrinos, which we assume to be massless, are relativistic. Use this to find the graviton temperature in terms of the present day photon temperature and further compute the number density of relic gravitons today. Again neglect the contribution of the graviton to g_{*s}. (*3 points*)
Hint: For the decoupled neutrinos you can use your result for $\frac{T_\nu}{T}$ for the electron-neutrinos from H.6.2 (d)

The above line of reasoning lets us estimate the graviton relic abundance based on general principles. In order to refine it, we would e.g. need to know if the graviton behaves like a relativistic degree of freedom at a certain temperature, which means we would need to know its mass. On the other hand the mass of the graviton depends on the explicit model for quantum gravity under consideration, which would require a separate less general analysis.