Physikalisches Institut
Universität Bonn
Theoretische Physik

Exercise 7
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Theoretical Particle Astrophysics

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https://www.th.physik.uni-bonn.de/people/berbig/SS20-TAPP/

Homework

Due July 1st, 2020

H.7.1 Quickies *(4 points)*

a) BBN can only explain the generation of light elements up to ^{7}Li. Briefly state how the heavier elements are produced. *(2 points)*

b) Explain what is meant with the 'deuterium bottleneck' of cosmological helium production. *(2 points)*

H.7.2 Dark Radiation during BBN *(12 points)*

Big Band Nucleosynthesis predicts the following abundance of Helium-4 nuclei

$$X_{^4\mathrm{He}} = \frac{4\; n_{^4\mathrm{He}}}{n_B} = \frac{2}{1 + \frac{n_p(T_{NS})}{n_n(T_{NS})}}, \tag{1}$$

where

$$\frac{n_n\left(T_{NS}\right)}{n_p\left(T_{NS}\right)} = \frac{n_n\left(T_n\right)\exp\left(-\frac{t_{NS}}{\tau_n}\right)}{n_p\left(T_n\right) + n_n\left(T_n\right)\left(1 - \exp\left(-\frac{t_{NS}}{\tau_n}\right)\right)} \tag{2}$$

and we introduce the neutron lifetime as $\tau_n = 886\,\mathrm{s}$. In this context we denote the neutron freeze-out temperature from the weak interaction as T_n. From the equilibrium discussion involving Saha equations we know that deuterium production becomes efficient at $T_{NS} \approx 80\,\mathrm{keV}$. The equilibrium number densities of the non-relativistic neutron (proton) are given by n_n (n_P).

a) Compute the age of the universe at T_{NS} by using that during radiation domination

$$t_{NS} = \frac{M^*_{\mathrm{Pl}}}{2\,T^2_{NS}} \quad \text{with} \quad M^*_{\mathrm{Pl}} = \frac{M_{\mathrm{Pl}}}{1.66\,\sqrt{g_*}} \tag{3}$$

holds true. The g_* which enters the total radiation energy density of the universe ($\propto T^4$) is given by

$$g_* = \sum_{\text{bosons}} g_i \left(\frac{T_i}{T}\right)^4 + \frac{7}{8}\sum_{\text{fermions}} g_i \left(\frac{T_i}{T}\right)^4 \tag{4}$$

and for a relativistic species in thermal equilibrium with the plasma we have $T = T_i$. *(2 points)*

b) The weak interaction rates involving neutrons can be estimated via dimensional analysis as

$$\Gamma_n = C_n\, G_F^2\, T^5. \tag{5}$$

$C_n = 1.2$ follows from nuclear physics and Fermi's constant reads $G_F = 1.17 \times 10^{-5}\,\mathrm{GeV}^{-2}$. By comparing this with the Hubble rate during radiation domination $H = \frac{T^2}{M_{\mathrm{Pl}}^*}$ find the neutron freeze out temperature T_n. [1] *(2 points)*

c) The contribution of any relativistic particle that is not a photon to the energy density of the universe $\rho_{\mathrm{rel.}}$ can be parameterized in terms of the effective number of neutrinos N_{eff} with temperature T_ν:

$$\rho_{\mathrm{rel.}} = \rho_\gamma \left(1 + \frac{7}{8}\left(\frac{T_\nu}{T_\gamma}\right)^4 N_{\mathrm{eff.}}\right) \tag{6}$$

In the Standard Model we have $N_{\mathrm{eff}} \approx 3$ [2] and the quantity $\Delta N_{\mathrm{eff.}} \equiv N_{\mathrm{eff.}} - 3$ is often used. Keep in mind that before electron positron-annhilations we have $T_\nu = T_\gamma$ and afterwards $\frac{T_\nu}{T_\gamma} = \left(\frac{4}{11}\right)^{\frac{1}{3}}$. If you were to assume the existence of dark radiation in the form of a new light particle in thermal equilibrium with the Standard Model plasma, briefly argue on a qualitative level how this would affect the above abundance of helium nuclei. *(2 points)*

d) In the previus parts you should have found that $T_n \approx 1.4\,\mathrm{MeV}$ and that $t_{NS} \approx 200\,\mathrm{s}$ leading to $X_{^4\mathrm{He}} \approx 45\%$. Now we want to estimate how changing $N_{\mathrm{eff.}}$ by a small amount $\Delta N_{\mathrm{eff.}}$ affects the helium abundance. To this end derive

$$\frac{\Delta X_{^4\mathrm{He}}}{X_{^4\mathrm{He}}} = -\frac{X_{^4\mathrm{He}}}{2}\left(\frac{\partial\left(\frac{n_p(T_{NS})}{n_n(T_{NS})}\right)}{\partial T_n}\frac{\mathrm{d}T_n}{\mathrm{d}N_{\mathrm{eff.}}} + \frac{\partial\left(\frac{n_p(T_{NS})}{n_n(T_{NS})}\right)}{\partial t_{NS}}\frac{\mathrm{d}t_{NS}}{\mathrm{d}N_{\mathrm{eff.}}}\right)\Delta N_{\mathrm{eff.}} \tag{7}$$

and find the coefficients of the previous expression. Show that $\Delta N_{\mathrm{eff.}} = 1$, which corresponds e.g. to a fourth neutrino generation in thermal equilibrium with the Standard Model plasma at BBN, causes a relative change in the helium abundance of about 5%. *(6 points)*

Since we have precise data on the cosmological abundances of light elements like $X_{^4\mathrm{He}} = 0.245$ [3] we can utilize the BBN prediction to constrain $N_{\mathrm{eff.}} = 2.88 \pm 0.16$ and thereby narrow down the properties of hypothetical new light particles.

[1] Note that the actual freeze out temperature is less than the result of this estimate, but we continue with it throughout this exercise due to its simplicity.

[2] It is not exactly 3 due to non-instantaneous decoupling effects.

[3] The difference to our estimate arises purely from the more accurate value for T_n.

H.7.3 Neutron burning *(4 points)*

The scattering rate for the reaction

$$p + n \leftrightarrow D + \gamma \tag{8}$$

is approximately given by

$$\Gamma_{p(n,\gamma)D} = n_p \cdot \langle \sigma |\vec{v}| \rangle_{p(n,\gamma)D} \quad \text{with} \quad \langle \sigma |\vec{v}| \rangle_{p(n,\gamma)D} \approx 6 \times 10^{-20}\,\mathrm{cm}^3\,\mathrm{s}^{-1}. \tag{9}$$

For the sake of simplicity we neglected the temperature dependence in the expression for the thermally averaged cross section times the relative velocity.

a) Express the proton number density in terms of the photon number density and $\eta_B = 6.2 \times 10^{-10}$. Estimate the freeze-out temperature by assuming the Hubble rate for a radiation dominated universe. What does this imply for the interaction at a temperature of $T_{NS} \approx 80\,\mathrm{keV}$? *(2 points)*

b) During which epoch does the reaction freeze out? Calculate the age of the universe at this temperature. *(2 points)*