Exercises General Relativity and Cosmology
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Hand in: Presence exercise

http://www.th.physik.uni-bonn.de/people/fierro/GRSS17/

−Class exercise−

1 Falling freely into a black hole (Extra 15 pts.)

Consider a particle that has crossed the event horizon of a Schwarzschild black hole at $r = R_S$ and is then falling freely within the event horizon. Work with the standard Schwarzschild coordinates (t, r, θ, ϕ) and denote the particle's proper time by τ.

1. Show that

$$\left| \frac{dr}{d\tau} \right| \geq \sqrt{\frac{R_S}{r} - 1} \,. \tag{1}$$

   When is this bound saturated?   (7* points)

2. Calculate the maximum lifetime of a particle falling freely from $r = R_S$ to $r = 0$. Plug in numbers to express this in seconds for a black hole with one solar mass, $M = 2 \cdot 10^{30}$kg.
                                                                   (4* points)

3. Can an observer that also travels from $r = R_S$ to $r = 0$ but not on a geodesic have a longer lifetime than the one calculated in item b)? Justify your answer.   (4* points)