Physikalisches Institut
Universität Bonn
Theoretische Physik

Exercise Sheet 5
14. May 2018
SS18

Exercises on Advanced Topics in String Theory

Priv.-Doz. Stefan Förste, Cesar Fierro, Urmi Ninad, Reza Safari

Hand in: 28.05.2018

More information at: `http://www.th.physik.uni-bonn.de/people/fierro/StringSS18/`

1 The Fermionic String Spectrum (15 Points)

In this exercise we aim to analyse the open and closed fermionic string spectrum upto to the first excited state, motivate the GSO projection and obtain the spacetime supersymmetric type-II string theories.

1. Worldsheet supersymmetry is achieved by adding fermions ψ to the worldsheet. The oscillator expansions of these new operators contain b_m modes, where $m \in \mathbb{Z} + \phi$ and $\phi = 0$ (1/2) for R (NS) boundary condition. The anti-commutation relations are given by:
$$\{b_r^\mu, b_s^\nu\} = \eta^{\mu\nu}\delta_{r+s} \tag{1}$$
The ground states of the Hilbert space now need to be distinguished depending on whether they lie in the R or NS sector. Define these respective ground states and explain why the Ramond vacuum has a degeneracy. (*2 points*)

2. For the *open* fermionic string, obtain the ground and the first excited state (and their $\alpha'(\text{mass})^2$) by acting the creation operators on the vacuum in the R and NS sector respectively.

   You will observe that the tachyonic mode appears again, but this time it can be consistently projected out using the so-called GSO projection. To do so, one first introduces an operator $(-1)^F$ (where F is the worldsheet fermion number) defined as :

   - NS sector: $(-1)^F = (-1)^{\sum_{r>0} b_r^i b_r^i - 1}$,
   - R sector: $(-1)^F = 16\ b_0^2 \dots b_0^9 (-1)^{\sum_{r>0} b_r^i b_r^i}$,

   acting on the respective vacua as:

   - $(-1)^F \left|0\right\rangle_{NS} = - \left|0\right\rangle_{NS}$
   - $(-1)^F \left|a\right\rangle_R = \left|a\right\rangle_R$; $(-1)^F \left|\dot{a}\right\rangle_R = - \left|\dot{a}\right\rangle_R$

   One then requires that the eigenvalue of $(-1)^F$ on every state in the Hilbert space be +1 in the NS-sector and +1 or -1 in the R-sector. Among one of the many benefits of such a projection is that it renders the spacetime spectrum supersymmetric (and of course discards the tachyon). (*3 points*)

3. The *closed* string spectrum requires both left and right movers. Obtain its spectrum at the massless level by taking a direct product of left and right movers (which could either in the be R or NS sector).

   In doing so, following information about the vector(V), spinor(S) and co-spinor(C) representations of the $SO(8)$ might come in handy:

$$
\begin{aligned}
\mathbf{8_V} \otimes \mathbf{8_V} &= \mathbf{1} \oplus \mathbf{28} \oplus \mathbf{35_V} \\
\mathbf{8_V} \otimes \mathbf{8_S} &= \mathbf{8_C} \oplus \mathbf{56_C} \\
\mathbf{8_V} \otimes \mathbf{8_C} &= \mathbf{8_S} \oplus \mathbf{56_S} \\
\mathbf{8_C} \otimes \mathbf{8_S} &= \mathbf{8_V} \oplus \mathbf{56_V} \\
\mathbf{8_C} \otimes \mathbf{8_C} &= \mathbf{1} \oplus \mathbf{28} \oplus \mathbf{35_V}
\end{aligned}
$$

   (*5 points*)

4. GSO projection can be applied individually to the holomorphic and anti-holomorphic part to obtain two (chirally) inequivalent configurations (the $\pm$ indicating the eigenvalue of $(-1)^F$):

   IIA:(NS+,NS+) $\oplus$ (R+,R$-$) $\oplus$ (NS+,R$-$) $\oplus$ (R+,NS+)

   IIB:(NS+,NS+) $\oplus$ (R+,R+) $\oplus$ (NS+,R+) $\oplus$ (R+,NS+)

   What is the massless spectrum for these two projections? (*5 points*)