Physikalisches Institut
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Theoretische Physik

Exercise Sheet 2
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Exercises Superstring Theory

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More information at:
http://www.th.physik.uni-bonn.de/people/fierro/StringWS1718/

1 Conserved charges L_f (10 points)

In the lecture you encountered (infinitely many) conserved charges corresponding to energy-momentum tensor conservation (in the conformal gauge),

$$L_f = 2T \int_0^l d\sigma\ f(\sigma^+)\ T_{++}(\sigma^+)\ ,$$

and likewise for the right-movers $\bar{L}_{\bar{f}}$.

(a) Show that for the open string there exists only one set of conserved charges: $L_f + \bar{L}_{\bar{f}}$. What (if any) conditions need to be imposed on the functions $f(\sigma^+)$ and $\bar{f}(\sigma^-)$ for this to happen? Are they Neumann or Dirichlet b.c dependent? (*4 points*)

(b) Show that the conserved charges L_f satisfy,

$$\Big\{L_f, X^\mu(\sigma,\tau)\Big\}_{P.B.} = 2\pi T\ f(\sigma^+)\ \partial_+ X^\mu(\sigma,\tau)\ .$$

In doing so, it might help to first compute:

$$\begin{aligned}\left\{\frac{1}{2\pi}\int d\sigma\ T_{01}\ ,\ X^\mu(\sigma,\tau)\right\}_{P.B.} &= \partial_\sigma X^\mu(\sigma,\tau)\ ,\\ \left\{\frac{1}{2\pi}\int d\sigma\ T_{00}\ ,\ X^\mu(\sigma,\tau)\right\}_{P.B.} &= \partial_\tau X^\mu(\sigma,\tau)\ .\end{aligned}$$

What does this tell you about the significance of L_f (in light of the transformation it induces on $X^\mu(\sigma,\tau)$)? (*6 points*)

2 Target space symmetries (5 points)

Having gained an insight into the local symmetries of the Polyakov action and the corresponding conserved currents, let's move on to the global (target space) symmetries and their currents i.e. the total momentum P^μ and the total angular momentum $J^{\mu\nu}$. Verify that these currents indeed generate the Poincaré algebra:

$$\begin{aligned}
\Big\{P^\mu \ , \ P^\nu\Big\}_{P.B.} &= 0 \ , \\
\Big\{P^\mu \ , \ J^{\rho\sigma}\Big\}_{P.B.} &= \eta^{\mu\sigma} P^\rho - \eta^{\mu\rho} P^\sigma \\
\Big\{J^{\mu\nu} \ , \ J^{\rho\sigma}\Big\}_{P.B.} &= \eta^{\mu\rho} J^{\nu\sigma} + \eta^{\nu\sigma} J^{\mu\rho} - \eta^{\nu\rho} J^{\mu\sigma} - \eta^{\mu\sigma} J^{\nu\rho} \ .
\end{aligned}$$