Physikalisches Institut
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Theoretische Physik

Exercise Sheet 4
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Exercises Superstring Theory

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More information at:
`http://www.th.physik.uni-bonn.de/people/fierro/StringWS1718/`

1 Propagator of the quantised bosonic string (10 points)

In the lecture you encountered special coordinates $(z, \bar{z})$, that were expressed as follows in terms of worldsheet coordinates (τ, σ):

$$(z, \bar{z}) = \Big(\exp\Big(\frac{2\pi\imath(\tau-\sigma)}{l}\Big), \exp\Big(\frac{2\pi\imath(\tau+\sigma)}{l}\Big)\Big) \ .$$

Note in particular that $\bar{z} \neq z^*$, unless a Wick rotation is performed: $\tau = \imath t$ $(t \in \mathbb{R})$.

(a) A quantum field theory requires an ordering prescription for the quantum operators. Physically interpret what happens to the time-ordering on the cylindrical worldsheet for the closed string for the new coordinates $(z, \bar{z})$. (*1 point*)

(b) Extend the propagator for the left-left sector that you saw in the class:

$$\langle X_L^\mu(\bar{z}) X_L^\nu(\bar{w})\rangle = \frac{1}{4}\alpha^{'}\eta^{\mu\nu}\ln\bar{z} - \frac{1}{2}\alpha^{'}\eta^{\mu\nu}\ \ln(\bar{z}-\bar{w}) \ ,$$

to rest of the permutations:

$$\begin{aligned}
\langle X_R^\mu(z) X_R^\nu(w)\rangle &= \frac{1}{4}\alpha^{'}\eta^{\mu\nu}\ln z - \frac{1}{2}\alpha^{'}\eta^{\mu\nu}\ \ln(z-w) \ , \\
\langle X_R^\mu(z) X_L^\nu(\bar{w})\rangle &= -\frac{1}{4}\alpha^{'}\eta^{\mu\nu}\ln z \ , \\
\langle X_L^\mu(\bar{z}) X_R^\nu(w)\rangle &= -\frac{1}{4}\alpha^{'}\eta^{\mu\nu}\ln\bar{z} \ ,
\end{aligned}$$

and finally the full propagator:

$$\langle X^\mu(z,\bar{z}) X^\nu(w,\bar{w})\rangle = -\frac{1}{2}\alpha^{'}\eta^{\mu\nu}\ \ln((z-w)(\bar{z}-\bar{w})) \ .$$

The information that might come in handy:

- Normal ordering for the zero modes is typically defined as:

$$: p^\nu x^\mu : = x^\mu p^\nu$$

whereas that for creation, annihilation operators $(m, n > 0)$,

$$: \alpha^\nu_m \alpha^\mu_{-n} : = \alpha^\mu_{-n} \alpha^\nu_m \ ,$$

and similarly for the $\bar{\alpha}$'s.

- The Taylor expansion:

$$\ln(1-x) = -\sum_{n=1}^{\infty} \frac{x^n}{n}$$

(*6 points*)

(c) Compare the full propagator to the Coulomb potential ϕ at position w of a charge q at z in 2d.
Prove that:

$$\vec{\nabla}^2 \langle X(z) X(w) \rangle \sim \delta^{(2)}(z-w) \ .$$

Note that the Laplacian of the Coulomb potential of a point charge behaves similarly, as is obvious from the (electrostatic) Maxwell's equation: $\vec{\nabla}^2 \phi \sim \rho$. (*3 points*)