Physikalisches Institut
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Theoretische Physik

Exercise Sheet 4
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Exercises Superstring Theory

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More information at:
http://www.th.physik.uni-bonn.de/people/fierro/StringWS1718/

1 The critical bosonic string (10 points)

The bosonic string can be quantised either using lightcone quantisation, i.e. fixing a gauge to impose the constraint equations, or using the manifestly Lorentz covariant path integral quantisation. Either of these methods can be used to solve for the normal ordering constant a and the critical dimension D of the bosonic string. In this exercise we will focus on the lightcone quantisation path.

(a) Why does an undetermined normal ordering constant show up at all in the quantisation procedure of the bosonic string? And why only one? (*2 points*)

(b) Using the fact that the origin of a can be be traced back to the Virasoro generator L_0,

$$(L_0 - a)\, |\phi\rangle_{\text{phys}} = 0 \ ,$$

write down a in terms of the critical dimension D of the bosonic string.
Hint: You might need to use the (curious) zeta function regularisation:

$$\zeta(s) = \sum_{n=1}^{\infty} n^{-s} \ ,$$

which can be analytically continued to:

$$\zeta(-1) = -\frac{1}{12} \ .$$

(*2 points*)

(c) The level matching condition for the closed string follows through in the lightcone quantisation procedure,

$$m^2 = \frac{4}{\alpha'}\Big(N - a\Big) = \frac{4}{\alpha'}\Big(\bar{N} - a\Big) \ ,$$

except this time only the *transverse* modes count. Calculate the mass of the zeroth and the first excited state of the closed bosonic string. (*3 points*)

(d) Using a degree of freedom argument, show that the first excited states must be massless. It might help to recall Wigner's classification of irreducible representations of the Poincaré group.

If you've done everything up to this point correctly, you will see that this condition fixes the D to 26 and hence the a to 1. What is the mass of the ground state? (*3 points*)

2 State degeneracy (5 points)

Because of the way the Fock space is built, there exists a certain degeneracy at each level of excitation. For simplicity (i.e lack of the level matching condition) we consider the open bosonic string with NN b.c. :

(a) Let there be a state $|M\rangle$, such that the number operator $\hat{N}$ acts on it as:

$$\hat{N}\,|M\rangle = M\,|M\rangle \ .$$

Show then that:

$$|M\rangle = \prod_{\mathrm{j}=1}^{k} \hat{\alpha}^{i_{\mathrm{j}}}_{-n_{\mathrm{j}}}\,|0,p\rangle$$

where,

$$M = \sum_{l=1}^{k} n_l \ .$$

(*2 points*)

(b) Show that the number of states $|M\rangle$ is d_M, is given by the coefficient of q^M in:

$$\prod_{n=1}^{\infty} (1-q^n)^{-24} \ .$$

(*3 points*)