Physikalisches Institut
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Theoretische Physik

Exercise Sheet 8
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WS 17/18

Exercises Superstring Theory

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More information at:
`http://www.th.physik.uni-bonn.de/people/fierro/StringWS1718/`

1 The conformal group in 2d (10 points)

In the previous sheet, you learnt about the conformal group in d-dimensions. In particular, for an infinitesimal coordinate transformation of the type:

$$x^\mu \to x^\mu + \epsilon^\mu \ ,$$

which results in the defining conformal transformation on the metric tensor, you derived a condition for the parameter ϵ^μ:

$$\partial_\mu \epsilon_\nu + \partial_\nu \epsilon_\mu = \frac{2}{d} \partial \cdot \epsilon \eta_{\mu\nu} \ .$$

This condition in 2d resembles the Cauchy-Riemann equations and by a suitable change of coordinates ,

$$z = x^1 + ix^2$$
$$\epsilon(z) = \epsilon^1 + i\epsilon^2 \ ,$$

the infinitesimal conformal transformations in 2d resemble a holomorphic coordinate transformation on z (and anti-holomorphic on $\bar{z}$ respectively):

$$z \to z + \epsilon(z) \ .$$

(a) Using the fact that $\epsilon(z)$ can be written as a Laurent series in z, show that the infinitesimal generators of the aforementioned (anti-)holomorphic transformations are:

$$l_m = -z^{m+1}\partial_z \ , \ \bar{l}_m = -\bar{z}^{m+1}\partial_{\bar{z}} \ .$$

What is their algebra? (*3 points*)

(b) Show that the only generators that are defined globally on the Riemann sphere are: l_{-1}, l_0 and l_1. What sort of transformations do each of them bring about on the coordinates? A look back at the second exercise of the previous sheet might help. These are the generators of the *global conformal group* in 2d. (*3 points*)

(c) The special linear group in 2d $SL(2,\mathbb{C})$ acts on the coordinates on the Riemann sphere as:

$$z \to z' = \frac{az+b}{cz+d} \ , \ \text{s.t. } ad - bc = 1$$

Show that the global conformal group in 2d is isomorphic to $SL(2,\mathbb{C})/\mathbb{Z}_2$. The $\mathbb{Z}_2$ simply accounts for the symmetry $(a,b,c,d) \to (-a,-b,-c,-d)$. (*3 points*)

2 The Virasoro Algebra (6 points)

The discussion of the previous exercise was entirely classical. In order to promote it to a quantum theory, we introduce an ordering to all operators in the theory and evaluate correlation functions of observables using operator product expansions. You saw the operator product expansion of the energy momentum tensor with itself in the lecture.

$T(z)$ can be written as a Laurent expansion in terms of the Virasoro modes:

$$T(z) = \sum_{n \in \mathbb{Z}} z^{-n-2} L_n \ .$$

Invert this relation to write L_n as a contour integral over a holomorphic function. Compute the algebra of the Virasoro generators using their form as a contour integral. (*6 points*)