Physikalisches Institut
Universität Bonn
Theoretische Physik

Exercise Sheet $3 + \frac{1}{2}$
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Exercises Superstring Theory

Priv.-Doz. Stefan Förste, Cesar Fierro, Urmi Ninad

-PRESENCE EXERCISE–

1 T is for Tension

In the following we discuss a first non-trivial example of the classical closed string. Let the X^μ fields be given by

$$X^0 = 2R\tau, \quad X^1 = R\sin 2\sigma, \quad X^2 = R\cos 2\sigma, \quad X^i = 0\,, \text{ for } i = 3, \ldots, d. \tag{1}$$

(a) Show that X^0 in (1) is indeed a solution of the closed bosonic string. Why are X^1 and X^2 not solutions of the equations of motion?

(b) Compute the total energy $P^0_{tot} = \int_0^\pi \mathrm{d}\sigma P^0$. What is the meaning of this result?

2 A speed of light rotating spaghetti stick

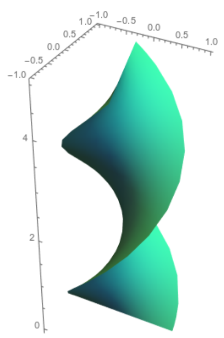

Figure 1: Worldsheet of the open string configuration given in (2). The vertical axis corresponds to the X^0 coordinate, which is orthogonal to the (X^1, X^2) plane.

Now consider the following open string rotating at a constant angular velocity in the (X^1, X^2) plane. Such a configuration is given by

$$X^0 = A\tau, \quad X^1 = A\cos\tau\cos\sigma, \quad X^2 = A\sin\tau\cos\sigma, \quad X^i = 0\,, \text{ for } i = 3, \ldots, d\,. \tag{2}$$

(a) Verify that (2) is indeed an open string configuration solution.

(b) Are the endpoints of the string satisfying Neumann or Dirichlet boundary conditions? What is the speed of the string at its endpoints?

(c) Compute the total energy $M \equiv P^0_{tot}$ of this configuration.

(d) Compute the angular momentum $J \equiv |J^{12}_{tot}|$, where $J^{\mu\nu}_{tot} = T\int_0^\pi d\sigma (X^\mu \dot{X}^\nu - X^\nu \dot{X}^\mu)$.

(e) Finally, compute $\alpha' \equiv \frac{J}{M^2}{}^2$[1].

[1] This parameter α' is the slope of the celebrated 'Regge' trajectories: the straight line plots of J vs. M^2 seen in nuclear physics in the late 1960s.