Physikalisches Institut
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Theoretische Physik

Exercise 01
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Exercises on General Relativity and Cosmology

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–Home Exercises–
Due 20 April 2011

Exercise 1.1: 4-Vectors (*3 credits*)

Consider any two non-zero orthogonal (with respect to the Minkowski metric) Lorentz 4-vectors $\vec{a}, \vec{b}$ which satisfy $\vec{a} \cdot \vec{b} = 0$.

(a) If $\vec{a}$ is space-like, show that $\vec{b}$ is necessarily time-like. (*2 credits*)

(b) Using a simple counter-example, show that a pair of non-zero orthogonal vectors does not necessarily mean that one is time-like and the other space-like. (*1 credit*)

Exercise 1.2: Metrics in different coordinate systems (*7 credits*)

(a) The measure of distance in flat three dimensional Euclidean space is $ds^2 = dx^2 + dy^2 + dz^2$. Write down the same in spherical polar coordinates $r,\ \theta,\ \phi$ defined by:

$$r = (x^2 + y^2 + z^2)^{1/2}\ ;\quad \cos(\theta) = z/r\ ;\quad \tan(\phi) = y/x$$

(*3 credits*)

(b) The measure of invariant distance in Minkowski space is $ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$. Find the components of the metric $g_{\mu\nu}$ in "rotating coordinates" defined by:

$$t' = t\ ;\quad x' = (x^2 + y^2)^{1/2}\ \cos(\phi - \omega t)\ ;\quad y' = (x^2 + y^2)^{1/2}\ \sin(\phi - \omega t)$$

(*4 credits*)

Exercise 1.3: Mikowski Diagrams (*10 credits*)

In this exercise we review spacetime diagrams. Note that here (and in everything that follows) $c = 1$, such that $[x] = [t]$.

(a) Use the spacetime diagram of an observer $\mathcal{O}$ to describe the following experiment performed by $\mathcal{O}$. Two bursts of particles of speed $v = 0.5$ are emitted from x=0 at t=-2m, one traveling in the positive x direction and the other in the negative x direction. These encounter detectors located at $x = \pm 2$m. After a delay of 0.5m of time, the detectors send sigmals back to x=0 at speed v=0.75. (*3 credits*)

(b) The signals arrive back at x=0 at the same event (make sure your spacetime diagram shows this). From this the experimenter concludes that the particle detectors did indeed send out their signals simultaneously, since he knows they are at equal distances from x=0. Explain why this conclusion is valid. (*2 credits*)

(c) A second observer $\overline{\mathcal{O}}$ moves with speed v=0.75 in the negative x direction relative to $\mathcal{O}$. Draw the spacetime diagram of $\overline{\mathcal{O}}$ and depict the experiment performed by $\mathcal{O}$. Does $\overline{\mathcal{O}}$ conclude that particle detectors sent out their signals simultaneously? If not, which signal was sent first? (*3 credits*)

(d) Compute the interval Δs^2 between the events at which the detectors emitted their signals, using both the coordinates of $\mathcal{O}$ and those of $\overline{\mathcal{O}}$. (*2 credits*)

Remark on parodoxes in special relativity:
While the good old "twin paradox" is quite easy to resolve (see e.g. spacetime diagrams here: http://www.csupomona.edu/~ajm/materials/twinparadox.html), "Bell's spaceship paradox" (see e.g. http://en.wikipedia.org/wiki/Bell's_spaceship_paradox) is harder.