Physikalisches Institut
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Theoretische Physik

Exercise 5
20 May 2015
SS 2015

Exercises on Advanced Topics in String Theory

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http://www.th.physik.uni-bonn.de/people/forste/exercises/strings15

–Home Exercises– Due to: 03.06.2015

H 5.1 Two point function for free fermion *(15 points)*

The action for a free Majorana fermion reads

$$S = \frac{1}{4\pi g}\int \mathrm{d}x^0\mathrm{d}x^1\sqrt{|h|}(-i)\overline{\Psi}\gamma^\alpha\partial_\alpha\Psi, \tag{1}$$

where g is a constant, $\overline{\Psi} = \Psi^\dagger\gamma^0$, $h_{\alpha\beta} = \mathrm{diag}(1,-1)$, and the gamma matrices are given by

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \gamma^0 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \tag{2}$$

(a) What is the Majorana condition on the components $\psi, \overline{\psi}$ of Ψ? (*2 points*)

(b) Perform a Wick rotation $x_1 \to ix_1$ and define $z := x^0 + ix^1$ to rewrite the action as

$$S = \frac{1}{4\pi g}\int \mathrm{d}z\mathrm{d}\overline{z}\left(\psi(z,\overline{z})\overline{\partial}\psi(z,\overline{z}) + \overline{\psi}(z,\overline{z})\partial\overline{\psi}(z,\overline{z})\right). \tag{3}$$

(*3 points*)

(c) Calculate the equation of motion for ψ and $\overline{\psi}$. What do they imply? (*1 point*)

(d) By imposing invariance of the action (3) under conformal transformations, calculate the conformal weighrs $(h, \overline{h})$ of ψ and $\overline{\psi}$. (*2 points*)

(e) Next we want to calculate the correlator $\langle\Psi_i(z,\overline{z}), \Psi_j(z',\overline{z}')\rangle$ where $i,j = 1,2$ label the comoponents of Ψ. To do so, express the kinetic terms of the components in (3) as a matrix A_{ij} and write down the differential equation for the Green's function. (*2 points*)

(f) We claim that the Green's function $G_{ij}(z,z')$ for the equation in (e) is given by

$$G = 2g\begin{pmatrix} \overline{\partial}\frac{1}{z-z'} & 0 \\ 0 & \partial\frac{1}{\overline{z}-\overline{z}'} \end{pmatrix} \tag{4}$$

Prove this by using the techniques you already learned for the bosonic case. (*5 points*)