Physikalisches Institut
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Theoretische Physik

Exercise 5
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SS 2015

Exercises on Advanced Topics in String Theory

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http://www.th.physik.uni-bonn.de/people/forste/exercises/strings15

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H 6.1 $SO(8)$ **spinor representation and triality** *(15 points)*

Superstring theory is only consistent in a spacetime of 10 dimensions. There will be a *Neveu-Schwarz* and *Ramond* sector for the mode expansion of the worldsheet fermions.

(a) What is the little group $G_{\text{little},10}$ for massless states in 10 dimensions with Poincare invariance? Explain why! (*1 point*)

(b) What is the dimension of a Dirac spinor of $G_{\text{little},10}$. (*1 point*)

For $SO(1,2n-1)$ the complex conjugate $T^{\mu\nu\,1}_{\pm}$ is given by

$$(T^{\mu\nu}_{\pm})^* = \begin{cases} -(C_{\pm}\gamma^0)T^{\mu\nu}_{\pm}(C_{\pm}\gamma^0)^{-1} & \text{for } n \text{ odd} \\ -(C_{\pm}\gamma^0)T^{\mu\nu}_{\mp}(C_{\pm}\gamma^0)^{-1} & \text{for } n \text{ even} \end{cases} . \tag{1}$$

(c) How many real degrees of freedom does a Weyl spinor of $SO(1,7)$ have? (*2 points*)

The Cartan matrix A_{ij} for $SO(8)$ is given by

$$A_{ij} = \begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & -1 \\ 0 & -1 & 2 & 0 \\ 0 & -1 & 0 & 2 \end{pmatrix} . \tag{2}$$

(d) Find the simple roots for $SO(8)$. (*2 points*)

(e) Calculate the states for the vector, spinorial, and co-spinorial representation. The highest weight of the vector represention (spinor / co-spinor) is given by $(1,0,0,0)$ ($(0,0,1,0)$ / $(0,0,0,1)$). What do the three representations have in common? (*4 points*)

(f) In superstring theory we have states from the Neveu-Schwarz sector and the Ramond sector. Why does the massless state $b^i_{-1/2}|0\rangle_{\text{NS}}$, with $i = 0,1,...,7$ in the Neveu-Schwarz sector transform in the vector representation of $SO(1,7)$ and why does the Ramond-ground state transform in the spinorial representation? *Hint: The Ramond-ground state is massless and* $\{b^i_0, b^i_0\} = \eta^{ij}$. (*3 points*)

[1] See exercise 3.2

(g) In order to construct a closed string theory we need to glue lefthanded states with righthanded states. Each state is transforms in one of the above calculated representations. Taking all possibilities into account how does the spectrum for massless states look like? Count the degrees of freedom for bosons and fermions and comment your result. *Hint: You might need the number of dimensions for the following $SO(8)$ representations:*

$$\begin{aligned}
\boldsymbol{8}_V \otimes \boldsymbol{8}_V &= \boldsymbol{1} \oplus \boldsymbol{28}_V \oplus \boldsymbol{35}_V \\
\boldsymbol{8}_V \otimes \boldsymbol{8}_S &= \boldsymbol{8}_C \oplus \boldsymbol{56}_C \\
\boldsymbol{8}_V \otimes \boldsymbol{8}_C &= \boldsymbol{8}_S \oplus \boldsymbol{56}_S \\
\boldsymbol{8}_C \otimes \boldsymbol{8}_S &= \boldsymbol{8}_V \oplus \boldsymbol{56}_V \\
\boldsymbol{8}_C \otimes \boldsymbol{8}_C &= \boldsymbol{1} \oplus \boldsymbol{28}_V \oplus \boldsymbol{35}_V
\end{aligned}$$

Where V denotes a vector representation, S a spinorial representation and C a cospinorial representation. (*2 points*)