Superstring Theory

Priv.-Doz. Dr. Stefan Förste und Christoph Nega

`http://www.th.physik.uni-bonn.de/people/forste/exercises/strings19`

Due date: 18.10.2019

–General information–

Responsible for the organization of the exercises is

Christoph Nega
Room W2.009 BCTP, Wegelerstraße 10
E-Mail: cnega@th.physik.uni-bonn.de

Please feel free to contact him in any case.

Your final grade will be based on your performance in the final exam. In order to be admitted to the final exam you have to collect 50% of the points from the exercise sheets. The exercise sheets will be collected every Friday during the lecture.

All information about the lecture, e.g. date of final exam, exercise sheets and others, can be found on the course's webpage

`http://www.th.physik.uni-bonn.de/people/forste/exercises/strings19`

The tutorials take place on

- Group 1: Tuesdays 8-10 in Room 2.008 BCTP by Andrei-Ioan Radoaca-Dogaru
- Group 2: Thursdays 8-10 in Room 2.008 BCTP by Mohamed Belhassen

–Class exercises–

1.1 The relativistic point particle

We consider the action for a relativistic point particle

$$S_{pp} = -m \int ds = -m \int \sqrt{-\eta_{\mu\nu}\dot{X}^\mu \dot{X}^\nu} d\tau \,,$$

where τ parameterizes the worldline of the particle and $\eta_{\mu\nu}$ is the d-dimensional Minkowski metric.

a) Show that the action S_{pp} is invariant under Poincaré transformations.

b) Show that the action S_{pp} is invariant under reparametrizations $\tau \mapsto \tau'(\tau)$.

c) Show that

$$p^\mu = \frac{m\dot{X}^\mu}{\sqrt{-\eta_{\nu\rho}\dot{X}^\nu\dot{X}^\rho}}$$

is a conserved quantity. Do this once by evaluating the Euler Lagrange equations and once by exploiting the symmetry $X^\mu \mapsto X^\mu + b^\mu$.

Hint: For the latter, assume that b^μ depends on τ to perform a partial integration and only then restrict to constant b^μ.

d) Argue that S_{pp} is inappropriate to describe massless particles.

e) Show that the new action

$$S_e = -\frac{1}{2}\int e\left(-\frac{1}{e^2}\dot{X}^\mu\dot{X}^\nu\eta_{\mu\nu} + m^2\right)d\tau$$

is equivalent to the action S_{pp}.

Hint: Integrate out e.

f) Explain the statement "We have coupled the particle to worldline gravity." and state the type of the field e.

g) Show that the new action S_e is invariant under reparametrizations of τ.

Hint: From f) you can refer how e transforms.

1.2 The Nambu-Gotu action versus the Polyakov action

The Nambu-Goto action for a string is given by

$$S_{NG} = -T\int d^2\sigma\sqrt{-\det\left(\partial_\alpha X^\mu\partial_\beta X^\nu\eta_{\mu\nu}\right)}\ ,$$

where σ^α with $\alpha = 0, 1$ label the worldsheet time τ and space σ.

a) Write down explicitly the action S_{NG} in terms of τ and σ.

b) Explain the geometric interpretation of the action S_{NG}.

c) Show that the Polyakov action

$$S_P = -\frac{T}{2}\int d^2\sigma\sqrt{-h}\left(h^{\alpha\beta}\partial_\alpha X^\mu\partial_\beta X^\nu\eta_{\mu\nu}\right)$$

is equivalent to the Nambu-Goto action S_{NG}.

Hint: The relation $\delta\sqrt{-h} = -\frac{1}{2}\sqrt{-h}h_{\alpha\beta}\delta h^{\alpha\beta}$ can be useful.

–HOMEWORKS–

1.3 Symmetries of the Polyakov action

We consider again the Polyakov action given by

$$S_P = -\frac{T}{2}\int d^2\sigma\sqrt{-h}\left(h^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu\nu}\right) \ .$$

In this exercise we want to focus on the symmetries of this action and the resulting implications. Before we start we recapitulate the Noether method which can be used to compute the conserved current associated to a symmetry of the action. Assume there is a Lagrangian which is invariant under an infinitesimal transformation of the fields defined by

$$\phi^a \mapsto \phi^a + \delta\phi^a \quad \text{with} \quad \delta\phi^a = \epsilon^i h_i^a(\phi^b) \ ,$$

where ϵ^i is infinitesimal and h_i^a denotes a function of the fields ϕ^a. Then the associated conserved current j_i^α is given by

$$\epsilon^i j_i^\alpha = \frac{\partial\mathcal{L}}{\partial(\partial_\alpha\phi^a)}\delta\phi^a \ .$$

Note that i might be a multi-index.

We start our discussion with an analysis of the **global symmetries** of the Polyakov action S_P.

a) Show that the Polyakov action S_P is invariant under Poincaré transformations given by

$$X^\mu \mapsto \Lambda^\mu{}_\nu X^\nu + b^\mu \ ,$$

with Λ a Lorentz transformation and b a shift. *(1 Point)*

b) Use the Noether method to compute the conserved currents associated to the Poincaré transformations.

<u>*Hint*:</u> The infinitesimal variations for Poincaré transformations are respectively given by

$$X^\mu \mapsto X^\mu + \epsilon a^\mu{}_\nu X^\nu \quad \text{with} \quad a_{\mu\nu} = -a_{\nu\mu} \qquad \text{and} \quad X^\mu \mapsto X^\mu + \epsilon^\mu \ .$$

Moreover, you can use the worldsheet metric $h_{\alpha\beta} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ to calculate the Noether currents. *(3 Points)*

Now we want to consider the **local symmetries** of S_P.

c) Show that the action S_P is invariant under worldsheet reparametrizations $\sigma^\alpha \mapsto \sigma'^\alpha(\sigma^\beta)$. *(1 Point)*

d) Moreover, show that S_P is invariant under Weyl transformations $h_{\alpha\beta} \mapsto \mathrm{e}^{\phi(\sigma^\alpha)} h_{\alpha\beta}$. *(1 Point)*

e) Argue that the local symmetries can be used to fix the worldsheet metric locally to

$$h_{\alpha\beta} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \ .$$

Give an argument why it is not possible in general to use this metric globally. *(3 Points)*

1.4 Two-dimensional gravity

Show that the energy momentum tensor in two dimensions vanishes identically due to the Einstein field equations.

<u>*Hint*:</u> Remember that due to the symmetries of the Riemann tensor there is only a single non-vanishing component in two dimensions. *(3 Points)*