Physikalisches Institut
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Theoretische Physik

Exercise Sheet 12
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WS 2019/20

Superstring Theory

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http://www.th.physik.uni-bonn.de/people/forste/exercises/strings19

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–Homeworks–

12.1 SO(9,1) representations vs. SO(8) representations and the superstring spectrum

Superstring theory is only consistent in a ten-dimensional spacetime. In ten dimensions the normal ordering constant is $-\frac{1}{2}$. There is a *Neveu-Schwarz* and a *Ramond* sector for the mode expansion of the worldsheet fermions. In this exercise we want to compute the spectrum of the superstring and see how it fits into $\mathrm{SO}(9,1)$ representations or $\mathrm{SO}(8)$ representations.

a) Write down the number of independent components a Dirac, a Weyl and a Majorana-Weyl spinor in ten dimensions has. State for which group the Majorana-Weyl spinors actually exist. *(2 Points)*

b) Determine the little group $G_{\text{little},10}$ for massless states in ten dimensions with Poincare invariance. *(1 Point)*

c) Write down the number of independent components a Dirac, a Weyl and a Majorana-Weyl spinor of $G_{\text{little},10}$ has. *(1 Point)*

We start with an analysis of the superstring vacuum. Thereby, one has to distinguish the Neveu-Schwarz and the Ramond ground states. The Neveu-Schwarz ground state is analogously defined as in the bosonic theory

$$\alpha_m^\mu \left|0\right\rangle_{\text{NS}} = b_r^\mu \left|0\right\rangle_{\text{NS}} = 0 \quad \text{for } m = 1,2,\dots \text{ and } r = \frac{1}{2},\frac{3}{2},\dots \; .$$

The Ramond ground state, as it turns out, is a spinor state satisfying

$$\alpha_m^\mu \left|a\right\rangle_{\text{R}} = b_m^\mu \left|a\right\rangle_R = 0 \quad \text{for } m = 1,2,\dots \; .$$

Now we want to argue why the Ramond ground state is a spinor and of what type it is.

d) Show that $b_0^\mu \left|0\right\rangle$ for $\mu = 0,1,\dots,9$ are degenerate in mass. *(2 Points)*

e) Explain why the states $b_0^\mu \left|0\right\rangle$ are spinors. Specify their corresponding group. *(1 Point)*

f) Compute the action of G_0 on $\left|a\right\rangle_R$. Explain its implications.
<u>*Hint*</u>: Recall the definition $G_r = \sum_{m\in\mathbb{Z}} \alpha_{-m}\cdot b_{r+m}$. *(1 Point)*

g) Compute the number of independent components the physical ground state $|a\rangle_R$ has. Compare this with the independent components a spinor of SO(8) has and interpret it.
Hint: Use the result from part f). (*2 Points*)

Having discussed the superstring vacuum we want to focus on the first excited states and construct from this a supersymmetric spectrum. We begin with the open string spectrum.

h) Determine the first few states in the Neveu-Schwarz sector up to $\alpha' m^2 = 1$. Moreover, show that they can be grouped in their corresponding little group representations. (*2 Points*)

i) Do the same for the Ramond sector. (*2 Points*)

j) Out of the so far described states pick a set of states forming a supersymmetric spectrum. Describe this procedure in terms of the GSO projection.
Hint: It might be useful to visualize the spectrum determined in h) and i) in a table. (*3 Points*)

Finally, we want to construct the closed superstring spectrum. This can be done by tensoring two open string spectra, one for the left- and one for the right-movers. This produces four different sectors being the (NS,NS)-, (R,R)-, (NS,R)- and the (R,NS)-sector.

k) State a condition the masses of the left- and right-movers have to satisfy to yield a candidate state in the superstring spectrum. (*1 Point*)

l) Determine the massless spectrum for closed strings.
Hint: You can use the following tensor products of SO(8) representations

$$\begin{aligned}
\mathbf{8}_V \otimes \mathbf{8}_V &= \mathbf{1} \oplus \mathbf{28}_V \oplus \mathbf{35}_V \\
\mathbf{8}_V \otimes \mathbf{8}_S &= \mathbf{8}_C \oplus \mathbf{56}_C \\
\mathbf{8}_V \otimes \mathbf{8}_C &= \mathbf{8}_S \oplus \mathbf{56}_S \\
\mathbf{8}_C \otimes \mathbf{8}_S &= \mathbf{8}_V \oplus \mathbf{56}_V \\
\mathbf{8}_C \otimes \mathbf{8}_C &= \mathbf{1} \oplus \mathbf{28}_V \oplus \mathbf{35}_V \\
\mathbf{8}_S \otimes \mathbf{8}_S &= \mathbf{1} \oplus \mathbf{28}_V \oplus \mathbf{35}_V \ ,
\end{aligned}$$

where V denotes a vector representation, S a spinorial representation and C a co-spinorial representation. (*3 Points*)

m) Pick again a set of the massless states which is supersymmetric. Comment whether this choice is unique or not und explain it in the context of the GSO projection. (*3 Points*)