Physikalisches Institut
Universität Bonn
Theoretische Physik

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WS 12/13

Übungen zu Theoretische Physik IV

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http://www.th.physik.uni-bonn.de/people/forste/exercises/ws1213/tp4

–Class Exercises–

A 8.1 Thermodynamical Inequalities

In the lecture we saw that one can deduce the inequalities

$$C_V \geq 0 \qquad \text{und} \qquad \kappa_T \geq 0$$

from the fact, that the variances of the statistical quanta E and V are positive. Here we want to derive these two *stability criteria* from macroscopic thermodynamics. Therefore, consider a system with energy E, volume V and particle number N. We segment this system into two equally sized subsystems and consider a change of energy E_1 of the first subsystem, its volume V_1 and its particle number N_1 by δE_1, δV_1 and δN_1 respectively. Consequently the energy E_2, volume V_2 and particle number N_2 of the second subsystem change by $\delta E_2 = -\delta E_1$, $\delta V_2 = -\delta V_1$ and $\delta N_2 = -\delta N_1$. The total entropy is given by

$$S(E, V, N) = S_1\left(\frac{E}{2}, \frac{V}{2}, \frac{N}{2}\right) + S_2\left(\frac{E}{2}, \frac{V}{2}, \frac{N}{2}\right) .$$

(a) Expand the variation of the entropy δS up to second order in δE_1, δV_1, δN_1. Which statement can you deduce from the stationarity of the entropy for temperatures, pressures and chemical potentials of the subsystems?

(b) Let $\delta N = 0$. Deduce the stability criteria from the maximality of the entropy.

Stability criteria of this kind are manifestations of the *principle of Le Chatelier*: If a system is in a stable equilibrium, every spontaneous change of its parameters leads to a reaction that brings the system back to the equilibrium.

A 8.2 Heating a Room

Here we want to calculate the heat needed to raise the temperature of a room from 0°C to 20°C. First we want to consider the isobaric process, that means that gas molecules can escape the room. Afterwards we calculate the heat needed for the case of a completely isolated room, that means for constant number of particles of the gas. We will assume the air inside the room to be a mixture of ideal gases of oxygen and nitrogen, such that $f = 5$.

(a) Calculate the heat needed to heat the room at constant pressure.

(b) Calculate the heat needed to heat the completely isolated room.

–Homework–

H 8.1 Richmannian Formula *5 Points*

An isolated container contains two materials which are separated by a fixed wall which is impermeable for particles but permeable for energy. In the beginning the materials have temperatures T_A and T_B and temperature independent heat capacities $C_{V,\mathrm{A}}$ and $C_{V,\mathrm{B}}$. Calculate the equilibrium temperature T_f of the system as well as its change of entropy. Is this process reversible?

H 8.2 Isothermals of the Van-der-Waals Gas *5 Points*

The equation of state of the Van-der-Waals gas which has been calculated in exercise H 4.1 is given by

$$\left(P + \frac{a}{v^2}\right)(v - b) = kT \,,$$

where $v = \frac{V}{N}$ is the volume per particle. Contrary to the ideal gas the isothermals in the p-V-plane of the Van-der-Waals gas are not monotonous functions for all values of T. Below a critical temperature T_c they have two extrema and, as one can see in figure 1, so called *Van-der-Waals-loops* form. Especially then there are regios in which $\left(\frac{\partial P}{\partial V}\right)_T > 0$ and the stability criterion $\kappa_T \geq 0$ is violated. Calculate the critial temperature T_c, for which the Isothermal only has a saddle point, while it has two extrema below and no extremum above this temperature.
Hint: Use a discriminant criterion and the Cardano's formulas.

H 8.3 Sound Waves in a Gas *(3+2+2+3=10) Points*

Here we want to consider the propagation of sound waves in a gas. If a sound wave propagates in a gas, the oscillation period is small compared to the relaxation time which a macroscopically small volume element needs, to exchange energy with the rest of the gas by heat transfer. Hence we can consider changes in density of such a volume element as adiabatic. We will restrict ourselves to the one-dimensional case. The equilibrium pressure of the gas is denoted by P_0.

(a) Consider a small slice as in figure 2, which, in equilibirum, has the width $\Delta x \ll 1$. Due to the sound wave the left side of the slice is deflected by ξ and the right side of the slice is deflected by $\xi + \Delta\xi$. Show the equation

$$\frac{\partial p}{\partial x} = -\rho \frac{\partial^2 \xi}{\partial t^2} \,,$$

where p is the the deviation of the pressure from the equilibrium pressure and ρ is the density of the gas, by looking at the forces which act on such a volume element. Here the extent of the volume element can be neglected.
Hint: You can approximate the change of pressure with x linearly.

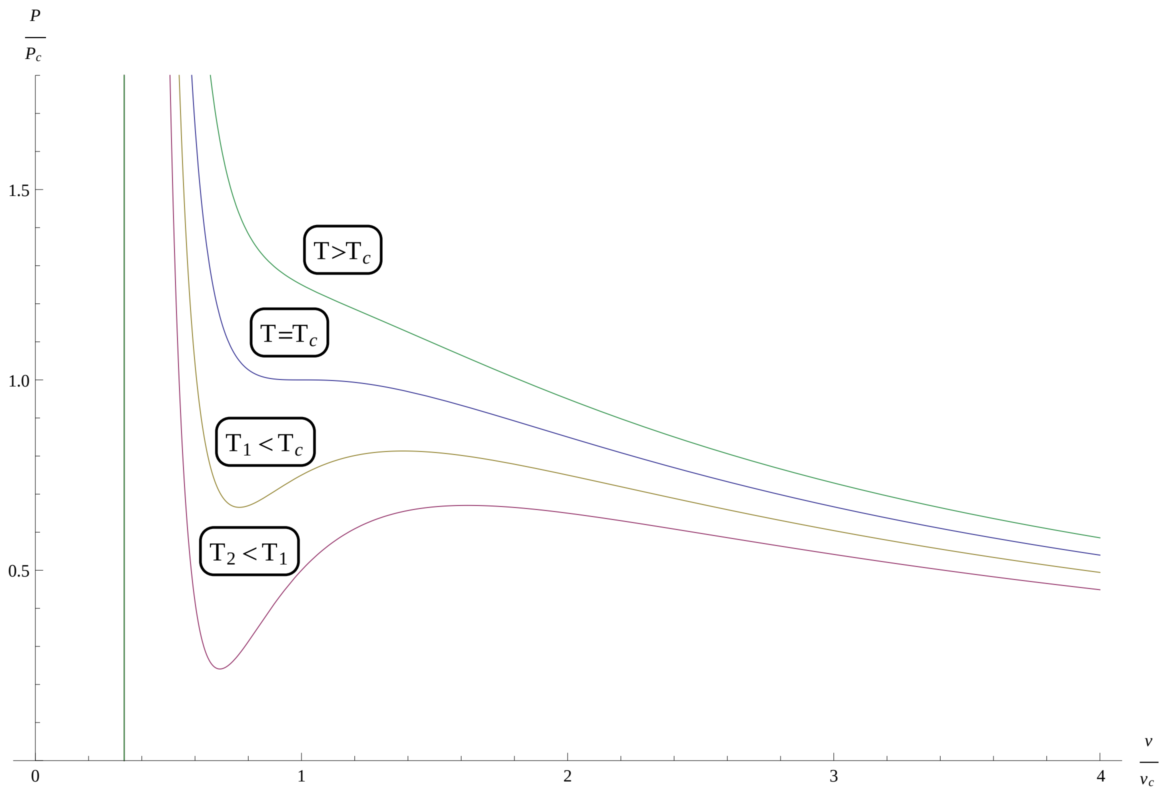

Figure 1: Isothermals of the Van-der-Waals gas in the p-V-plane for different values of T.

(b) Show, that in the limit $\Delta x \to 0$,

$$p = -\frac{1}{\kappa_S}\frac{\partial \xi}{\partial x}$$

where the variation of p with x can be neglected. Use this result to show

$$\frac{\partial^2 p}{\partial t^2} = \frac{1}{\kappa_S \rho}\frac{\partial^2 p}{\partial x^2}\,,$$

that means that the change of pressure fulfills a wave equation with the speed of sound $u = (\rho\kappa_S)^{-1/2}$.

(c) Express the adiabatic bulk modulus κ_S of an ideal gas in terms of its pressure and the ratio $\gamma = \frac{C_p}{C_V}$.

(d) Express the speed of sound in the ideal gas in terms of γ, the molar mass μ and the temperature T. How does the speed of sound depend on the temperature for fixed pressure? How does it depend on the equilibrium pressure P_0 for fixed temperature? *Hint: Use the equation of state of the ideal gas.*

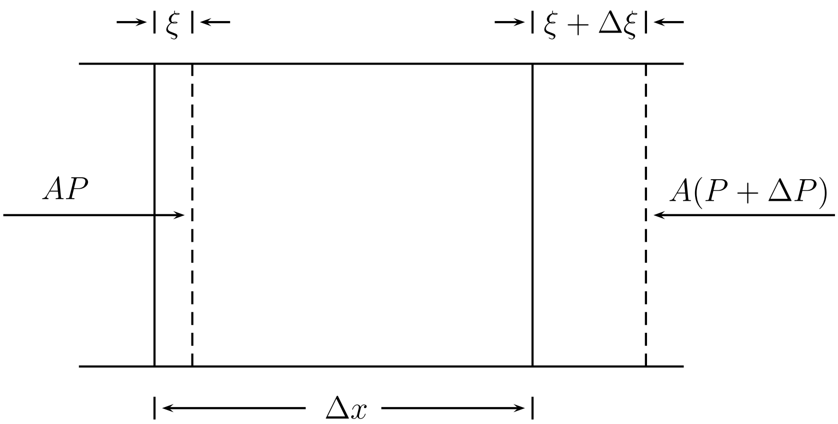

Figure 2: Illustration for the sound wave in a gas.