Universität Bonn 08.12.2005
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Condensed Matter Theory I — WS05/06

Exercise 5

(Please return your solutions before 20.12., 13:00 h)

5.1 Quasiparticle distribution function (6 points)

In this exercise we want to study how the presence of an interaction changes the quasiparticle momentum distribution function $n_{k,\sigma}$.

a) Calculate the free distribution function $n^0_{k,\sigma} = \langle a^\dagger_k a_k \rangle^0$ from the bare Green's function

$$G^0_{k,\sigma}(\omega) \;=\; \frac{1}{i\omega - \varepsilon_k}$$

in the limit $T \to 0$. Discuss the difference of $n_{k,\sigma}$ with respect to the Fermi-Dirac distrubution.

b) Show that the distribution function from a) has a discontinuity at $k = k_F$ and calculate the size of this step.

c) Now consider the renormalised Green's function from exercise 4.2

$$G_{k,\sigma}(\omega) \;=\; \frac{z}{i\omega - (\varepsilon^*_k - \mu^*)} \;+\; G^{incoh}_{k,\sigma}(\omega)$$

The incoherent part $G^{incoh}_{k,\sigma}(\omega)$ contains the higher order corrections around the pole and is a continous function at $k = k_F$. It has been neglected in exercise 4.2. Do the same calculations as for the free Green's function, 5.1 a), b), and compare the result with the one from above.

5.2 Quasiparticle lifetime (10 points)

In the lecture the concept of quasiparticles was introduced. In this exercise we will find a microscopic reason why this is in normal Fermi liquids an appropriate description at least in the case of low-temperature excitations. Therefore we will calculate the lifetime of these excitations in the limit $\omega \to 0$, $T = 0$.

a) What is the connection between the imaginary part of the single particle self energy and the lifetime τ of an excitation?

b) In first order perturbation theory the self energy is always real as we saw in exercise 4.5. Therefore we have to do a second order calculation now. Consider again the local potential from exercise 3.3 and show that the self energy corresponding to the Feynman diagram of Fig. 1 is given by:

$$\Sigma_{k,\sigma}(\omega) = -2V_0^2 \sum_{k',k''} \sum_{\omega_n} \frac{f(\varepsilon_{k''})\,(1 - f(\varepsilon_{k'+k''}))}{i\omega - i\omega_n - \varepsilon_{k-k'}} \cdot \left(\frac{1}{i\omega_n + \varepsilon_{k''} - \varepsilon_{k'+k''}} - \frac{1}{i\omega_n - \varepsilon_{k''} + \varepsilon_{k'+k''}} \right)$$

c) Perform the limit $T \to 0$ and the continuation to real frequencies and show that the imaginary part of the self energy is given by:

$$\mathrm{Im}\,(\Sigma_{k,\sigma}(\omega)) = 2V_0^2 \sum_{\substack{k',\,k'' \\ \varepsilon_{k''} < 0 < \varepsilon_{k'} \\ \varepsilon_{k+k''-k'} > 0}} \delta(\omega - \varepsilon_{k'} + \varepsilon_{k''} - \varepsilon_{k+k''-k'}) - \sum_{\substack{k',\,k'' \\ \varepsilon_{k''} < 0 < \varepsilon_{k'} \\ \varepsilon_{k+k''-k'} < 0}} \delta(\omega + \varepsilon_{k'} - \varepsilon_{k''} - \varepsilon_{k+k''-k'})$$

d) In general the convolution of the momenta prohibits the calculation of the remaining sums. For an general estimate we will therefore neglect the momentum conservation and rewrite the last result as

$$\langle \mathrm{Im}\,(\Sigma_\sigma(\omega)) \rangle_k \sim V_0^2 \sum_{\substack{k',k'',k''' \\ \varepsilon_{k''} < 0 < \varepsilon_{k'}, \varepsilon_{k'''}}} \delta(\omega - \varepsilon_{k'} + \varepsilon_{k''} - \varepsilon_{k'''}) - \sum_{\substack{k',k'',k''' \\ \varepsilon_{k''}, \varepsilon_{k'''} < 0 < \varepsilon_{k'}}} \delta(\omega + \varepsilon_{k'} - \varepsilon_{k''} - \varepsilon_{k'''})$$

Assume that the density of states is bounded ($N(\varepsilon) \leq N_0$). Show that we can estimate the imaginary part of the self energy by

$$\mathrm{Im}\,(\Sigma_{k,\sigma}(\omega)) \sim V_0^2 N_0^3 \omega^2$$

e) Use the results of a) and d) to show $1/\tau \sim \omega^2$. What does this result mean for the validity of the quasiparticle concept? Discuss under which conditions the behavior $1/\tau \sim \omega^2$ can break down.

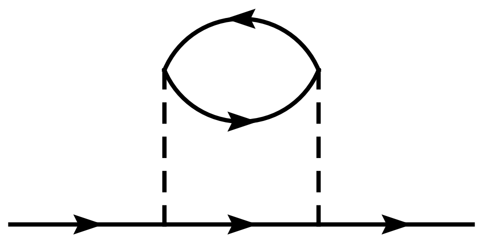

Figure 1: Second Order Self Energy Contribution