Efficient Calculation of Gluon Amplitudes
A Comparison

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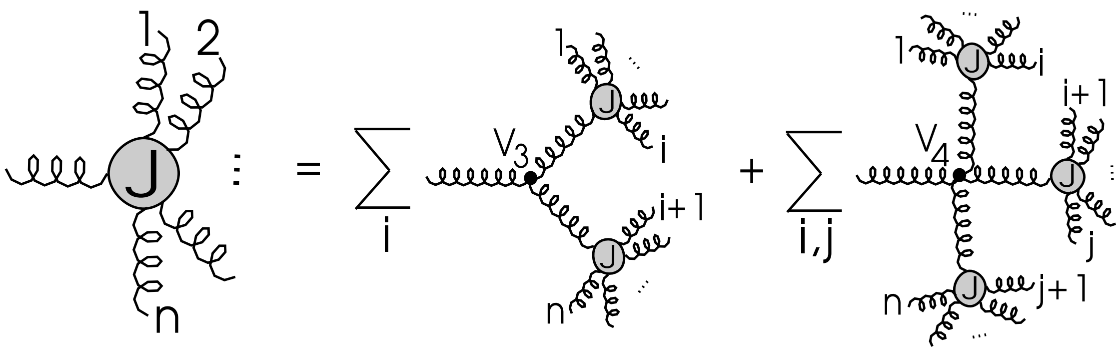


in collaboration with Michael Dinsdale and Stefan Weinzierl

Contents

- Why are n-gluon tree graphs interesting?
- How to calculate them classically?
- Can we do this any better?
- Berends & Giele recurrence relations
- Recursive calculation with scalar diagrams
- Recursive calculation with MHV vertices
- Recursive calculation with shifted momenta
- Summary & Outlook

Why are n-gluon tree graphs interesting?

- Estimate QCD backgrounds to new physics
- multijet events at hadron and e^+e^- colliders
- amongst parton processes gluons play a special role, since they have the largest parton cross section

How to calculate them classically?

- Feynman (1950s): draw all possible diagrams, use Feynman rules to get a formula, calculate the amplitude
- Drawbacks
  - too many diagrams, for example $gg \to 8g$ 10 million diagrams
  - too many terms in each diagram
  - too many kinematic variables
- intermediate expressions are vastly more complicated than the final result

Can we do this any better?

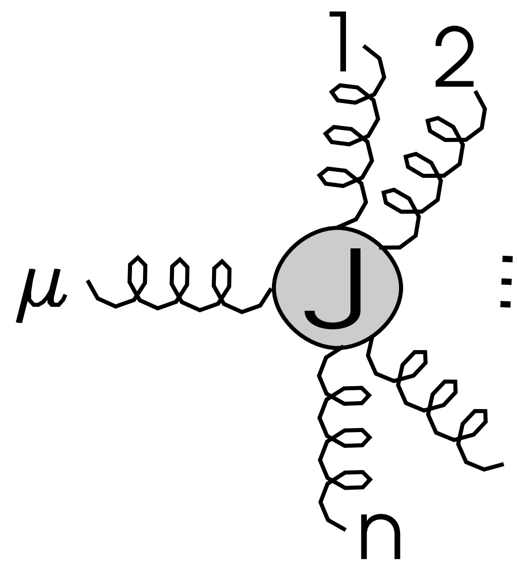
- Yes we can!
  - color decomposition

$$\mathcal{A}_n\left(\{k_i, \lambda_i, a_i\}\right) = g^{n-2} \sum_{\sigma \in S_n/Z_n} \mathrm{Tr}\left(T^{a_{\sigma(1)}} \cdots T^{a_{\sigma(n)}}\right) A_n(\sigma(k_1^{\lambda_1}), \ldots, \sigma(k_n^{\lambda_n}))$$

  - spinor helicity formalism
    introduce a new set of kinematic objects (spinor products)
  - use recursive relations to reduce the computational complexity
- Advantages
  - don't care about millions of diagrams, because recursion takes automatically into account all Feynman diagrams
  - significantly more efficient than conventional Feynman rules

Berends & Giele recurrence relations I

Nucl. Phys. B306 (1988) 759



- start from $A_n(k_1^{\lambda_1}, \ldots, k_n^{\lambda_n}, k_{n+1}^{\lambda_{n+1}})$
- remove the polarization vector ε^μ of gluon (n+1)
- add a propagator $\frac{-ig\mu\nu}{k_{n+1}^2}$

Berends & Giele recurrence relations I

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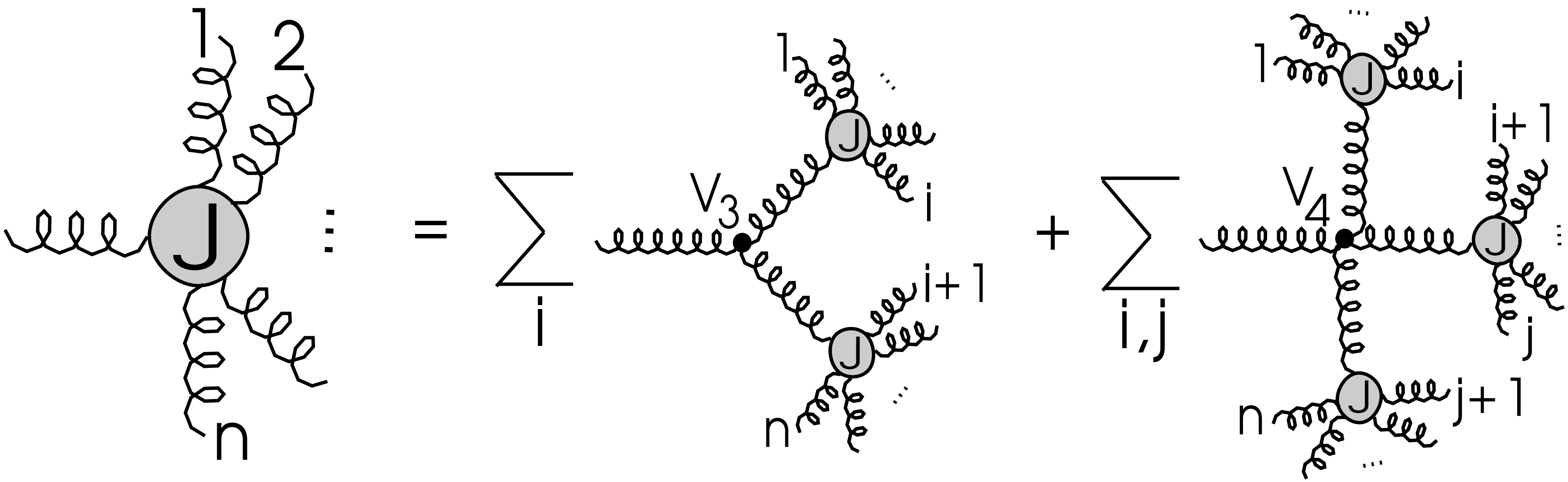

Berends & Giele recurrence relations I

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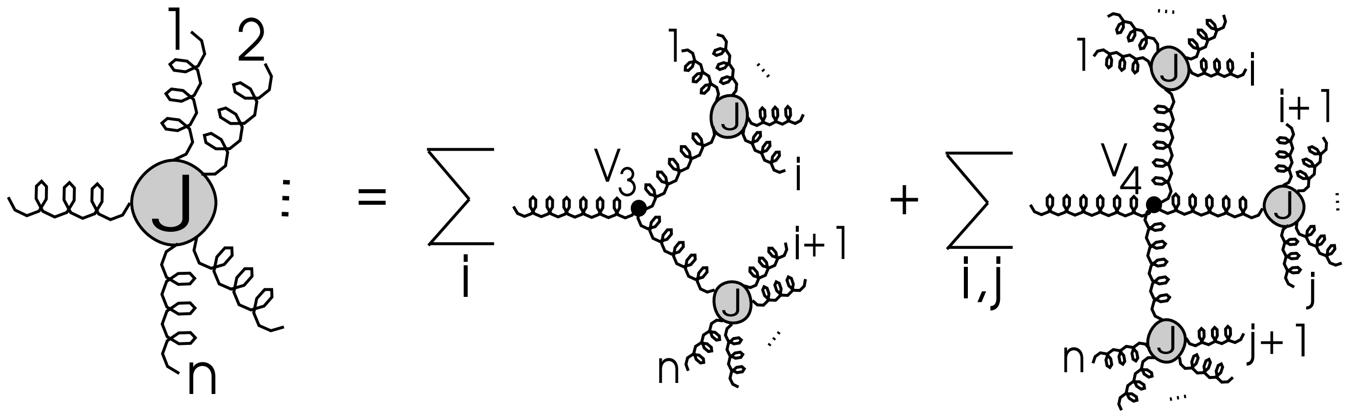

$$J^\mu(k_1^{\lambda_1},\ldots,k_n^{\lambda_n}) \;=\; \frac{-i}{P_{1,n}^2}\left[\sum_{i=1}^{n-1} V_3^{\mu\nu\rho}(P_{1,i},P_{i+1,n})\,J_\nu(k_1^{\lambda_1},\ldots,k_i^{\lambda_i})\,J_\rho(k_{i+1}^{\lambda_{i+1}},\ldots,k_n^{\lambda_n})\right.$$

$$\left.+\sum_{i=1}^{n-2}\sum_{j=i+1}^{n-1} V_4^{\mu\nu\rho\sigma}\,J_\nu(k_1^{\lambda_1},\ldots,k_i^{\lambda_i})J_\rho(k_{i+1}^{\lambda_{i+1}},\ldots,k_j^{\lambda_j})J_\sigma(k_{j+1}^{\lambda_{j+1}},\ldots,k_n^{\lambda_n})\right]$$

$$V_3^{\mu\nu\rho}(P,Q) \;=\; i\left(g^{\nu\rho}(P-Q)^\mu + 2g^{\rho\mu}Q^\nu - 2g^{\mu\nu}P^\rho\right)$$

$$V_4^{\mu\nu\rho\sigma} \;=\; i\left(2g^{\mu\rho}g^{\nu\sigma} - g^{\mu\nu}g^{\rho\sigma} - g^{\mu\sigma}g^{\nu\rho}\right)$$

$$P_{i,j} \;=\; \sum_{l=i}^{j} k_l$$

Berends & Giele recurrence relations II

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- starting point

$$J^\mu(k_i^{\lambda_i}) \quad = \quad \varepsilon^\mu(k_i^{\lambda_i}, q) \quad = \quad \pm\frac{\langle q^\mp|\gamma^\mu|k_i^\mp\rangle}{\sqrt{2}\langle q^\mp|k_i^\pm\rangle}$$

- Parke-Taylor amplitudes can be confirmed numerically

$$\begin{aligned} A_n(k_1^+, k_2^+, k_3^+, \ldots, k_n^+) &= 0 \\ A_n(k_1^-, k_2^+, k_3^+, \ldots, k_n^+) &= 0 \\ A_n(k_1^-, k_2^-, k_3^+, \ldots, k_n^+) &\neq 0 \end{aligned}$$

the last one is called maximally helicity violating or MHV amplitude

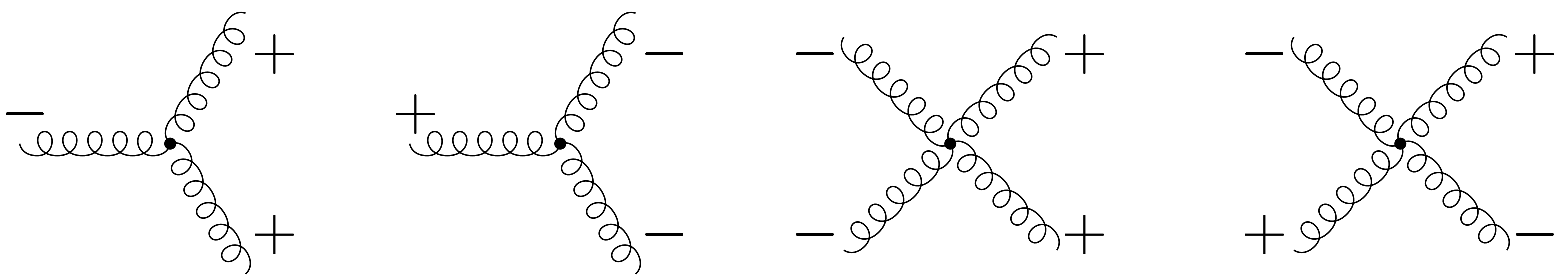
Recursive calculation with scalar diagrams

Schwinn & Weinzierl hep-th/0503015

- off-shell amplitude

$$
\begin{aligned}
O_n(k_1^{\lambda_1},\ldots,k_n^{\lambda_n}) \quad = \quad & \sum_{\lambda,\lambda'=\pm}\sum_{j=2}^{n-1} V_3(K_{1,j-1}^{\lambda},K_{j,n-1}^{\lambda'},k_n^{\lambda_n}) \\
\times \frac{i}{K_{1,j-1}^2} O_j(k_1^{\lambda_1},\ldots,k_{j-1}^{\lambda_{j-1}},-K_{1,j-1}^{-\lambda}) & \frac{i}{K_{j,n-1}^2} O_{n-j+1}(k_j^{\lambda_j},\ldots,k_{n-1}^{\lambda_{n-1}},-K_{j,n-1}^{-\lambda'}) \\
+ \sum_{\lambda,\lambda',\lambda''=\pm}\sum_{j=2}^{n-2}\sum_{l=j+1}^{n-1} & V_4(K_{1,j-1}^{\lambda},K_{j,l-1}^{\lambda'},K_{l,n-1}^{\lambda''},k_n^{\lambda_n}) \frac{i}{K_{1,j-1}^2} O_j(k_1^{\lambda_1},\ldots,k_{j-1}^{\lambda_{j-1}},-K_{1,j-1}^{-\lambda}) \\
\times \frac{i}{K_{j,l-1}^2} O_{l-j+1}(k_j^{\lambda_j},\ldots,k_{l-1}^{\lambda_{l-1}},-K_{j,l-1}^{-\lambda'}) & \frac{i}{K_{l,n-1}^2} O_{n-l+1}(k_l^{\lambda_l},\ldots,k_{n-1}^{\lambda_{n-1}},-K_{l,n-1}^{-\lambda''})
\end{aligned}
$$

- starting point $O_2(k_{j-1}^{\lambda},-K_{j-1,j}^{-\lambda}) \quad = \quad -ik_{j-1}^2$



Recursive calculation with MHV vertices

Cachazo, Svrček & Witten hep-th/0403047 Bena, Bern & Kosower hep-th/0406133

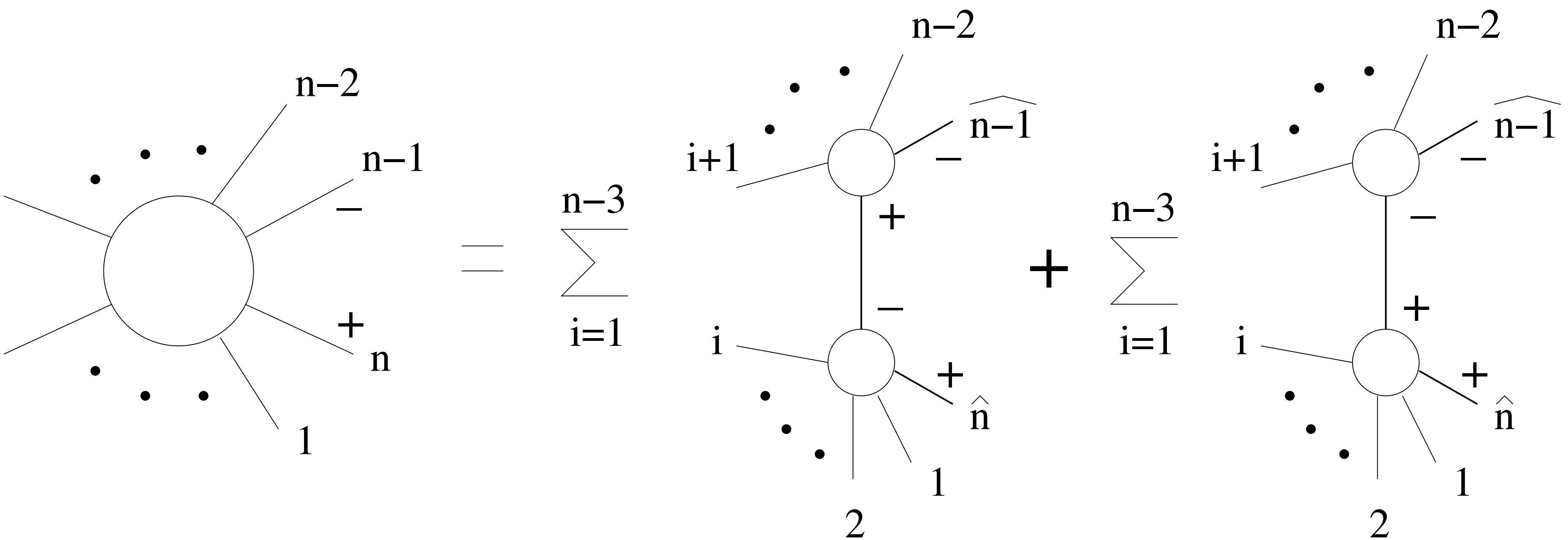
$$V_n(k_1^{\lambda_1},\ldots,k_n^{\lambda_n}) = \frac{1}{(n_{neg}-2)}\sum_{j=1}^{n}\sum_{l=j+1}^{j-3}\frac{i}{K_{j,l}^2}$$
$$\times V_{(l-j+2)\bmod n}(k_j^{\lambda_j},\ldots,k_l^{\lambda_l},-K_{j,l}^-)V_{(j-l)\bmod n}(k_{l+1}^{\lambda_{l+1}},\ldots,k_{j-1}^{\lambda_{j-1}},-K_{l+1,j-1}^+)$$

- starting point

$$V_n(k_1^+,\ldots,k_j^-,\ldots,k_k^-,\ldots,k_n^+) = i(\sqrt{2})^{n-2}\frac{\langle jk\rangle^4}{\langle 12\rangle\ldots\langle n1\rangle}$$
$$V_n(k_1^-,\ldots,k_j^+,\ldots,k_k^+,\ldots,k_n^-) = i(\sqrt{2})^{n-2}\frac{[kj]^4}{[1n][n(n-1)]\ldots[21]}$$

Recursive calculation with shifted momenta

Britto, Cachazo & Feng hep-th/0412308



Recursive calculation with shifted momenta

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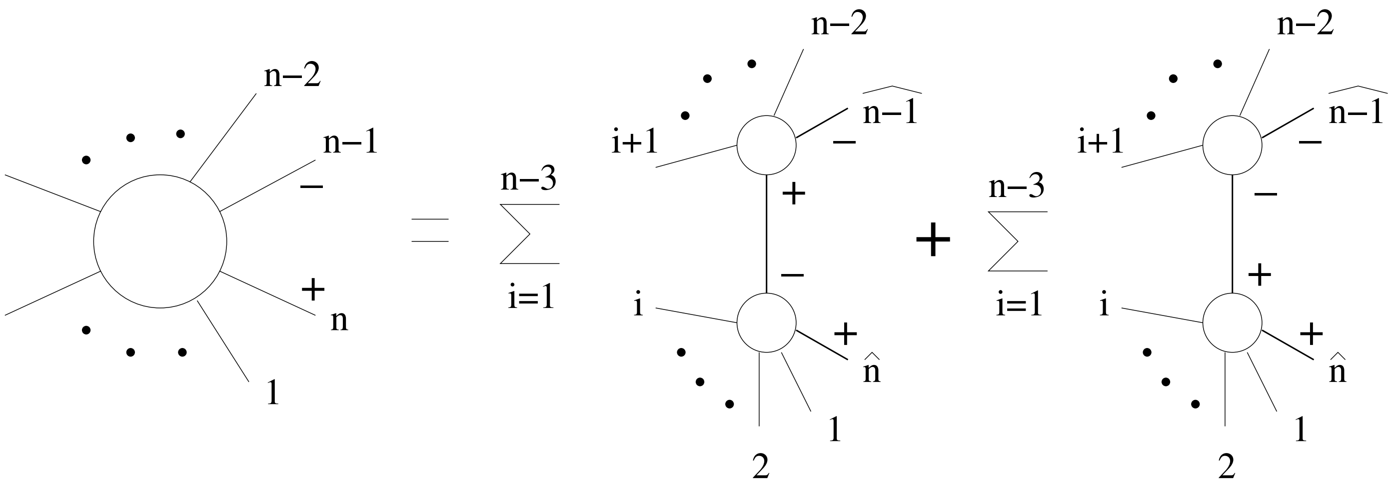

shifted momentum

$$A_n(k^1_A, k^1_{\dot B}, \lambda_1, \ldots, k^n_A, k^n_{\dot B}, \lambda_n) \quad =$$

$$\sum_{j=3}^{n-1} \sum_{\lambda=\pm} A_j \left(\hat{k}^1_A, k^1_{\dot B}, \lambda_1, k^2_A, k^2_{\dot B}, \lambda_2, \ldots, k^{j-1}_A, k^{j-1}_{\dot B}, \lambda_{j-1}, i\hat{K}_A, i\hat{K}_{\dot B}, -\lambda \right)$$

$$\times \frac{i}{K^2_{1,j-1}} A_{n-j+2} \left(\hat{K}_A, \hat{K}_{\dot B}, \lambda, k^j_A, k^j_{\dot B}, \lambda_j, \ldots, k^{n-1}_A, k^{n-1}_{\dot B}, \lambda_{n-1}, k^n_A, \hat{k}^n_{\dot B}, \lambda_n \right)$$

Summary & Outlook

- efficient methods for computing n-gluon tree graphs were presented
- especially different recursion relations
  - Berends & Giele recurrence relations
  - Recursive calculation with scalar diagrams
  - Recursive calculation with MHV vertices
  - Recursive calculation with shifted momenta
- Berends-Giele recursion relation has been implemented
- implement the other recursion relations
- compare the run time