Inclusive $\bar{B} \to X_s \ell^+ \ell^-$ in the Standard Model

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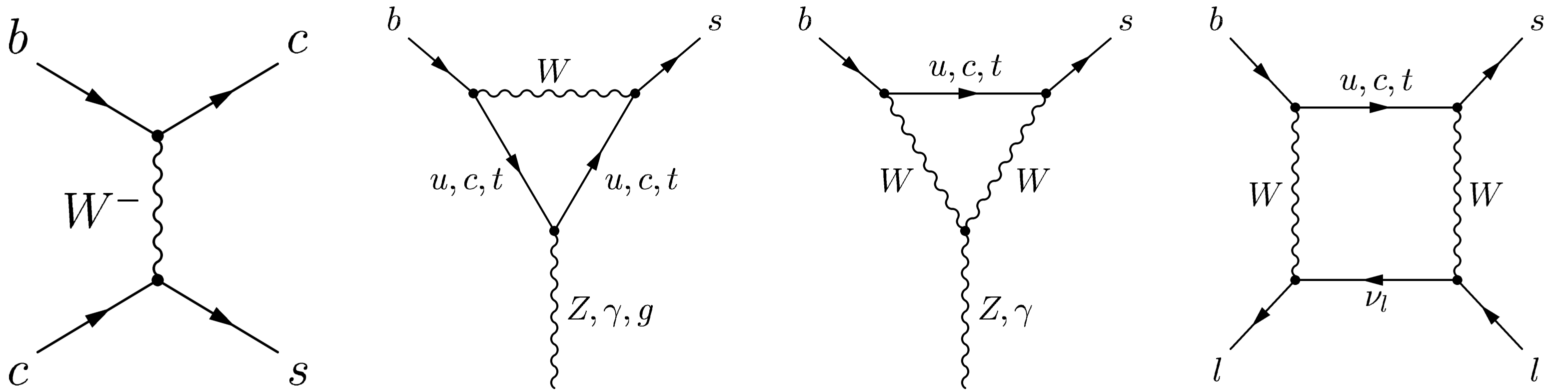
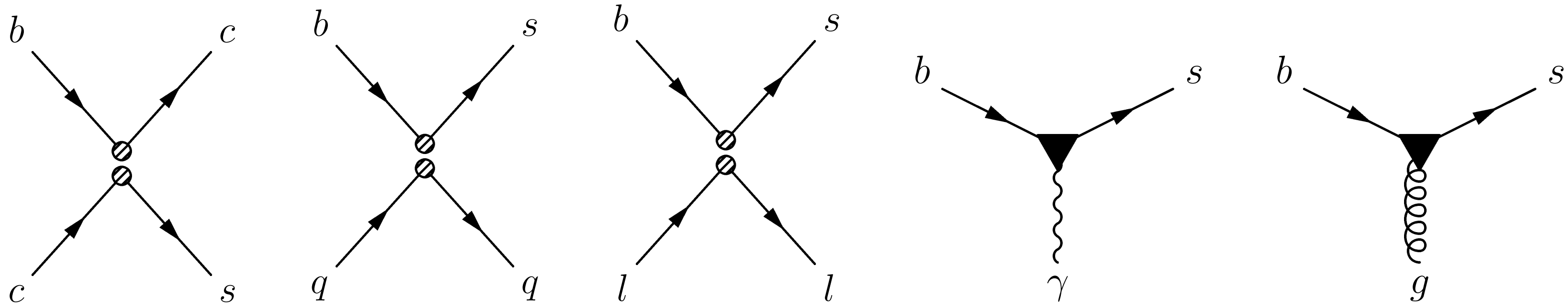
Effective theory of $\Delta B = 1$ decays

Standard Model $\Delta B = 1$ decays involve mass hierarchies:



(ext. momenta, $m_s, m_b \ll M_W, M_Z, m_t, M_{\rm NP}$)

“integrating out” heavy degrees of freedom $\Rightarrow$ Effective Theory (ET) :



List of all operators of QCD & QED analysis (in the SM)

$\Delta B = 1$ hadronic and radiative decays

$$Q_1 = (\bar{s}_L \gamma_\mu T^a c_L)(\bar{c}_L \gamma^\mu T^a b_L)\,, \qquad Q_2 = (\bar{s}_L \gamma_\mu c_L)(\bar{c}_L \gamma^\mu b_L)\,,$$

$$Q_3 = (\bar{s}_L \gamma_\mu b_L)\textstyle\sum_q (\bar{q}\gamma^\mu q)\,, \qquad Q_4 = (\bar{s}_L \gamma_\mu T^a b_L)\textstyle\sum_q (\bar{q}\gamma^\mu T^a q)\,,$$

$$Q_5 = (\bar{s}_L \gamma_\mu \gamma_\nu \gamma_\rho b_L)\textstyle\sum_q (\bar{q}\gamma^\mu \gamma^\nu \gamma^\rho q)\,, \qquad Q_6 = (\bar{s}_L \gamma_\mu \gamma_\nu \gamma_\rho T^a b_L)\textstyle\sum_q (\bar{q}\gamma^\mu \gamma^\nu \gamma^\rho T^a q)\,,$$

$$Q_3^Q = (\bar{s}_L \gamma_\mu b_L)\textstyle\sum_q Q_q (\bar{q}\gamma^\mu q)\,, \qquad Q_4^Q = (\bar{s}_L \gamma_\mu T^a b_L)\textstyle\sum_q Q_q (\bar{q}\gamma^\mu T^a q)\,,$$

$$Q_5^Q = (\bar{s}_L \gamma_\mu \gamma_\nu \gamma_\rho b_L)\textstyle\sum_q Q_q (\bar{q}\gamma^\mu \gamma^\nu \gamma^\rho q)\,, Q_6^Q = (\bar{s}_L \gamma_\mu \gamma_\nu \gamma_\rho T^a b_L)\textstyle\sum_q Q_q (\bar{q}\gamma^\mu \gamma^\nu \gamma^\rho T^a q)\,,$$

$$Q_1^b = -\tfrac{1}{3}(\bar{s}_L \gamma_\mu b_L)(\bar{b}\gamma^\mu b) + \tfrac{1}{12}(\bar{s}_L \gamma_\mu \gamma_\nu \gamma_\rho b_L)(\bar{b}\gamma^\mu \gamma^\nu \gamma^\rho b)\,,$$

$$Q_7 = \tfrac{e}{g_s^2} m_b (\bar{s}_L \sigma^{\mu\nu} b_R) F_{\mu\nu}\,, \qquad Q_8 = \tfrac{1}{g_s} m_b (\bar{s}_L \sigma^{\mu\nu} T^a b_R) G^a_{\mu\nu}\,,$$

additionally for $b \longrightarrow sll$

$$Q_9 = \tfrac{e^2}{g_s^2}(\bar{s}_L \gamma_\mu b_L)\textstyle\sum_\ell (\bar{\ell}\gamma^\mu \ell)\,, \qquad Q_{10} = \tfrac{e^2}{g_s^2}(\bar{s}_L \gamma_\mu b_L)\textstyle\sum_\ell (\bar{\ell}\gamma^\mu \gamma_5 \ell)\,.$$

Problem: radiative corrections (in PT) $\longrightarrow$ $\boxed{\alpha^n \ln^n(m/M) = \alpha^n [\ln(m/\mu_0) + \ln(\mu_0/M)]^n}$

$\longrightarrow$ bad convergence of perturbative expansion

Solution: 1. effective theory (OPE) without heavy degrees of freedom, available energy in B-decays

$$\sqrt{s} \sim 5\text{GeV} \ll M_W, M_Z, m_t, M_{\text{NP}}$$

2. renormalization group (RG) improvement $\longrightarrow$ resummation of large logarithms

Result: separation of short- and long-distance interactions of the full theory into effective coupling constants C_i and effective couplings of light particles $\mathcal{O}_i \longrightarrow \mathcal{L}_{\text{eff}}$

- C_i ... Wilson coefficients – short distance part
- $\mathcal{O}_i$ ... operators (flavor changing couplings)

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{QED}\times\text{QCD}}(u, d, s, c, b, e, \mu, \tau) + \frac{4G_F}{\sqrt{2}} V_{\text{CKM}} \sum_{\text{SM}} C_i \mathcal{O}_i + \sum_{\text{NP}} C_j \mathcal{O}_j$$

Determination of Wilson coefficients

In PT $G_F,\ \alpha_s G_F,\ \alpha_s^2 G_F,\ \alpha_e G_F,\ \alpha_e \alpha_s G_F$

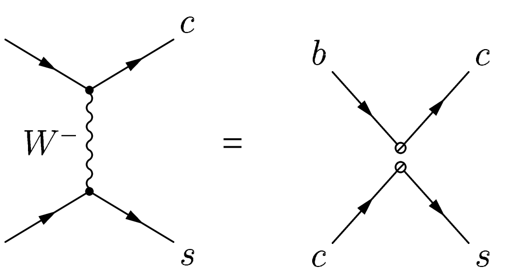
$$\boxed{\mathcal{A}^{\text{eff}}(m, C_i, \mu_0) \stackrel{!}{=} \mathcal{A}^{\text{full}}(m, M, \mu_0)} \quad \text{Matching}$$

- m-dependence equal, because effective theory $\stackrel{!}{=}$ full theory in the infrared $\longrightarrow$ $\ln(m/\mu_0)$ cancel and $C = C(M, \mu_0)$ independent of m
- must set $\mu_0 \approx M \longrightarrow \ln(\mu_0/M) \approx 0 \longrightarrow$ convergence of PT $\Rightarrow$ Matching scale μ_0

Wilson coefficients are determined at the scale μ_0!

$$C(\mu_0) = C_s^{(0)}(\mu_0) + \frac{\alpha_s(\mu_0)}{4\pi} C_s^{(1)}(\mu_0) + \frac{\alpha_s(\mu_0)^2}{(4\pi)^2} C_s^{(2)}(\mu_0) + \frac{\alpha_e}{4\pi} C_e^{(1)}(\mu_0) + \frac{\alpha_e \alpha_s(\mu_0)}{(4\pi)^2} C_e^{(2)}(\mu_0) + \ldots$$

Tree-level Matching:



[Buchalla/Buras], [Misiak/Urban], [Bobeth/Misiak/Urban], [Buras/Gambino/Haisch], [Gambino/Haisch]

Renormalization Group Equation (RGE)

$$\left(\mu \frac{d}{d\mu} \mathbb{1} - \hat{\gamma}^T\right) \vec{C}(\mu) = 0$$ Running

Anomalous Dimension Matrix (ADM)

$$\gamma = \frac{\alpha_s}{4\pi}\gamma_s^{(0)} + \frac{\alpha_s^2}{(4\pi)^2}\gamma_s^{(1)} + \frac{\alpha_s^3}{(4\pi)^3}\gamma_s^{(2)} + \frac{\alpha_e}{4\pi}\gamma_e^{(1)} + \frac{\alpha_e \alpha_s}{(4\pi)^2}\gamma_e^{(2)}$$

$$\vec{C}(\mu_b \approx m) = \hat{U}(\mu_b, \mu_0, \alpha_e)\vec{C}(\mu_0)$$

Resummation of large logarithms to all orders n

- LO $\longrightarrow$ $\alpha_s^n \ln^n(\mu_b/\mu_0)$
- NLO $\longrightarrow$ $\alpha_s^n \ln^{n-1}(\mu_b/\mu_0)$
- NNLO $\longrightarrow$ $\alpha_s^n \ln^{n-2}(\mu_b/\mu_0)$ $\Rightarrow$ $\mathcal{L}_{\text{eff}}^{\text{NNLO}}(\mu_b)$
- QED LO $\longrightarrow$ $\alpha_e \ln(\mu_b/\mu_0)\ \alpha_s^{n-1} \ln^n(\mu_b/\mu_0)$
- QED NLO $\longrightarrow$ $\alpha_e \ln(\mu_b/\mu_0)\ \alpha_s^{n-1} \ln^{n-1}(\mu_b/\mu_0)$

QCD: [Chetyrkin/Misiak/Münz], [Gambino/Gorbahn/Haisch], [Gorbahn/Haisch],
QED: [Baranowski/Misiak], [Bobeth/Gambino/Gorbahn/Haisch], [Huber/Lunghi/Misiak/Wyler]

Inclusive $\bar{B} \to X_s \ell^+ \ell^-$ - Heavy Quark Expansion

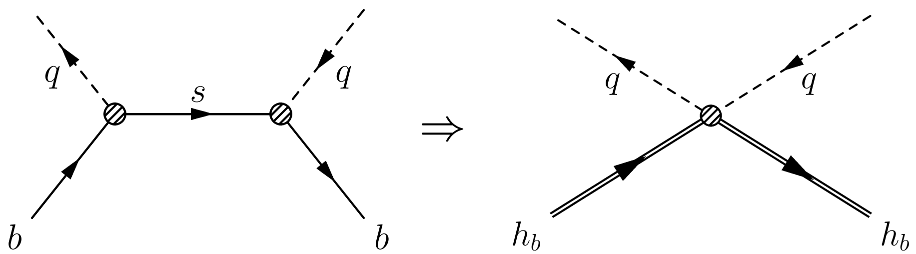
$$d\Gamma = \frac{1}{2M_B} \sum_{X_s} d[PS](2\pi)^4 \delta^{(4)}(p_B - p_{X_s} - q) \langle B | i\mathcal{L}_{\rm eff}^\dagger | X_s l^+ l^- \rangle \langle l^+ l^- X_s | i\mathcal{L}_{\rm eff} | B \rangle$$

$\Downarrow$ optical theorem $\longrightarrow$ absorptive part of $B \to B$

$$d\Gamma \sim \frac{1}{2M_B} d[PS](2\pi)^4 \delta^{(4)}(p_B - p_{X_s} - q) \; {\rm Im} \langle B | \hat{T}\{\mathcal{L}_{\rm eff}^\dagger \mathcal{L}_{\rm eff}\} | B \rangle$$

local OPE + HQET matrix elements – z_i Wilson coefficients ($\mu \sim m_b \gg \Lambda_{\rm QCD}$)

$$\Rightarrow \hat{T}\{\mathcal{L}_{\rm eff}^\dagger \mathcal{L}_{\rm eff}\} \sim z_1 (\bar{b}b) + \frac{z_2}{m_b^2} (\bar{b} g \sigma \cdot G b) + \sum \frac{z_{qi}}{m_b^3} (\bar{b} \Gamma_i q)(\bar{q} \Gamma_i b) + \ldots$$



+ higher dimension

$$\Rightarrow \langle B|\bar{b}b|B\rangle = 1 + \frac{1}{2m_b^2}\langle B|\bar{h}_b(iD)^2 h_b|B\rangle + \frac{1}{4m_b^2}\langle B|\bar{h}_b(g\sigma\cdot G)h_b|B\rangle + \ldots$$

$$= 2M_B\left[1 + \frac{1}{2m_b^2}(\lambda_1 + 3\lambda_2)\right] + \mathcal{O}\left[\left(\frac{\Lambda_{\mathrm{QCD}}}{m_b}\right)^3\right]$$

$\lambda_1 = (-0.3 \pm 0.1)\mathrm{GeV}^2, \quad \lambda_2 = 0.12\mathrm{GeV}^2$

$$d\Gamma \sim \text{parton} \;\; \text{result} + \mathcal{O}\left[\left(\frac{\Lambda_{\mathrm{QCD}}}{m_b}\right)^2\right]$$

$\Lambda_{\mathrm{QCD}}/m_b$ corrections known up to 3rd order

[Chay/Georgi/Grinstein], [Falk/Luke/Savage], [Ali/Hiller], [Buchalla/Isidori], [Bauer/Burrell]

$\Rightarrow$ similar power corrections $\Lambda_{\mathrm{QCD}}/m_c$ are known up to 2nd order

[Buchalla/Isidori/Rey]

matrix elements at parton level (virtual + real corrections)

- QCD:

$$(1+\frac{\alpha_s}{4\pi}M_s^{(1)}+\frac{\alpha_s^2}{(4\pi)^2}M_s^{(2)}) < \mathcal{O} >_{\rm tree}$$

Almost complete except NNLO matrix elements of $Q_3 - Q_6$ (small Wilson coefficients)

[Asatrian/Asatrian/Greub/Walker], [Ghinculov/Isidori/Hurth/Yao]

- QED:

$$(1+\frac{\alpha_e}{4\pi}M_e^{(1)}) < \mathcal{O} >_{\rm tree}$$

including collinear logarithms: $\ln(m_l/m_b)$

[Huber/Lunghi/Misiak/Wyler]

Differential Decay Rate

$$\frac{d\Gamma[\bar{B}\to X_s\ell^+\ell^-]}{d\hat{s}} = \frac{G_F^2 m_{b,\rm pole}^5\,|V_{ts}^*V_{tb}|^2}{48\pi^3}\left(\frac{\alpha_e(\hat{s})}{4\pi}\right)^2(1-\hat{s})^2\left\{\left(4+\frac{8}{\hat{s}}\right)\left|\widetilde{C}_{7,\rm BR}^{\rm eff}(\hat{s})\right|^2\right.$$

$$\left.+(1+2\hat{s})\left(\left|\widetilde{C}_{9,\rm BR}^{\rm eff}(\hat{s})\right|^2+\left|\widetilde{C}_{10,\rm BR}^{\rm eff}(\hat{s})\right|^2\right)+12\,{\rm Re}\left(\widetilde{C}_{7,\rm BR}^{\rm eff}(\hat{s})\widetilde{C}_{9,\rm BR}^{\rm eff}(\hat{s})^*\right)+\frac{d\Gamma^{\rm Brems}}{d\hat{s}}\right\}$$

Experimental information

- $q^2 \equiv (p_{l^-} + p_{l^+})^2 \ldots$ invariant mass of the l^+l^--pair
- $M_{X_s} \ldots$ invariant mass of the final hadronic state X_s

Integrated $d\mathcal{B}/dq^2 \times 10^{-6}$

$q^2 \in [q^2_{min}, q^2_{max}]$	Belle	BaBar	Average
$q^2 \in [(2m_\mu)^2, (m_B - m_K)^2]\text{GeV}^2$	$4.11 \pm 0.83^{+0.74}_{-0.70}$	$5.6 \pm 1.5 \pm 0.6 \pm 1.1$	4.5 ± 1.0
$q^2 \in [1, 6]\text{GeV}^2$	$1.49 \pm 0.50^{+0.38}_{-0.28}$	$1.8 \pm 0.7 \pm 1.5$	1.6 ± 0.5

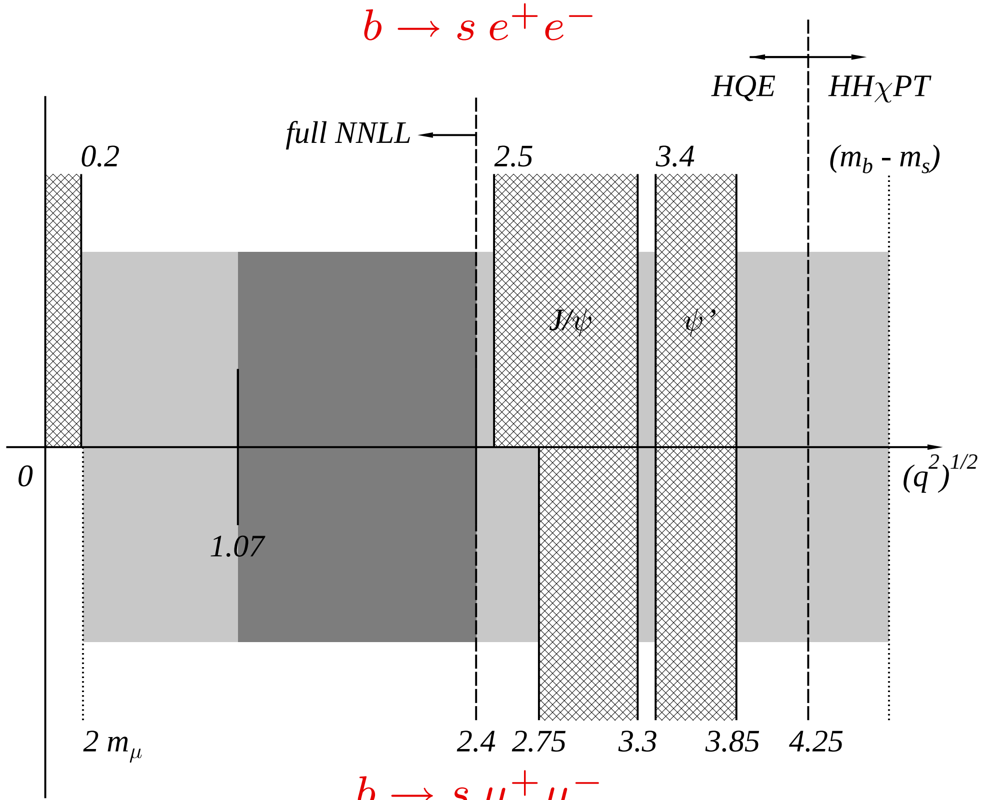
Cuts in

- $M_{X_s} \ldots$: $M_{X_s} < 2\text{GeV}$ [Belle] and $M_{X_s} < 1.8\text{GeV}$ [BaBar]
  extrapolation beyond cut with Fermi-Motion model [Ali/Hiller]
- $q^2 \ldots$: $(2m_\mu)^2 < q^2$ and “around charm-resonances”

Experimental Cuts in q^2 example [Belle]

because of background from Charm-Resonances: $b \to X_s c\bar{c} \to X_s l^+ l^-$

$b \to s\, e^+ e^-$



$b \to s\, \mu^+ \mu^-$

Theoretical predictions work best in the low-q^2 region!

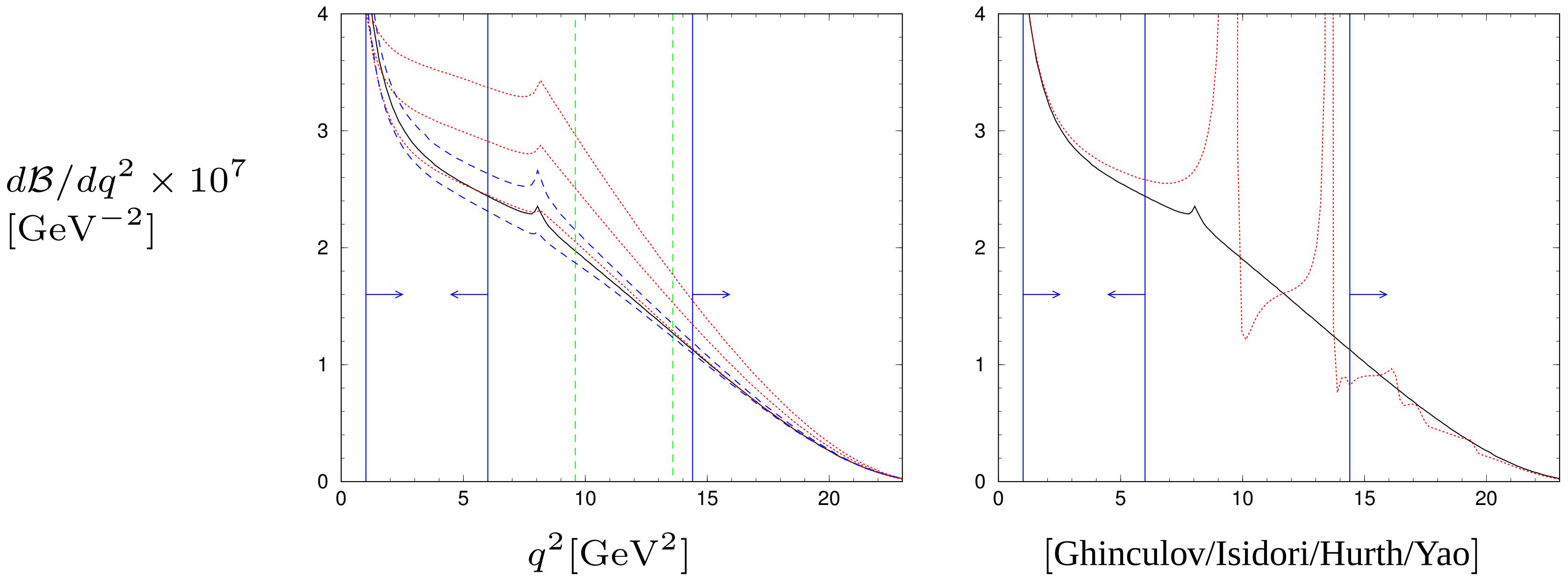
Low q^2-region: $q^2 \in [1, 6]\mathrm{GeV}^2, \quad \sqrt{q^2} \in [1.07, 2.4]\mathrm{GeV}.$

NNLO QCD corrections

- change of the differential $\mathcal{B}(\bar{B} \to X_s \ell^+ \ell^-)$ by -20% (-25%) in low-q^2 (high-q^2) region
- reduction of scale uncertainties (μ_0 and μ_b) from $\pm 20\%$ ($\pm 15\%$) to $\pm 6\%$ ($\pm 3\%$) in low-q^2 (high-q^2) region

NLO QED corrections

- inclusion of NLO QED corrections $\longrightarrow$ reduction of uncertainty of $\pm 8\%$ due to choice of $\alpha_e(m_b) \sim 1/133$ or $\alpha_e(M_W) \sim 1/128$ at LO in QED ($\mathcal{B}(\bar{B} \to X_s \ell^+ \ell^-) \sim \alpha_e^2$)
- collinear logs: enhancement of $\bar{B}(B \to X_s \mu^+ \mu^-)$ by $+2\%$ and $\bar{B}(B \to X_s e^+ e^-)$ by $+5\%$



[Ghinculov/Isidori/Hurth/Yao]

Final theoretical result for $q^2 \in [1,6]\mathrm{GeV}^2$ region (without M_{X_s} cuts)

$$\mathcal{B}(B \to X_s e^+ e^-) = \Big[1.64 \pm 0.08\big|_{\rm scale} \pm 0.06\big|_{m_t} \pm 0.025\big|_{C,m_c} \pm 0.015\big|_{m_b} \pm 0.02\big|_{\alpha_s(M_Z)} \pm 0.015\big|_{\rm CKM} \pm 0.026|_{{\rm BR}_{sl}}\Big] \times 10^{-6}$$

$$\mathcal{B}(B \to X_s \mu^+ \mu^-) = \Big[1.59 \pm 0.08\big|_{\rm scale} \pm 0.06\big|_{m_t} \pm 0.024\big|_{C,m_c} \pm 0.015\big|_{m_b} \pm 0.02\big|_{\alpha_s(M_Z)} \pm 0.015\big|_{\rm CKM} \pm 0.026|_{{\rm BR}_{sl}}\Big] \times 10^{-6}$$

Combining uncertainties in quadrature

$$\mathcal{B}(B \to X_s e^+ e^-) = (1.64 \pm 0.11) \times 10^{-6}$$
$$\mathcal{B}(B \to X_s \mu^+ \mu^-) = (1.59 \pm 0.11) \times 10^{-6}$$

Agrees with experimental world average of

$$\mathcal{B}(\bar{B} \to X_s \ell^+ \ell^-) = (1.6 \pm 0.5) \times 10^{-6}$$

Standard Model in aggreement with experimental data

???no new physics in $\bar{B} \to X_s \ell^+ \ell^-$ in low-q^2 region???

Experimental Cuts in M_{X_s}: to remove backgrounds - for example $b \to c(\to se^+\nu)e^-\bar{\nu}$

$M_{X_s} < 2\text{GeV}$ [Belle] and $M_{X_s} < 1.8\text{GeV}$ [BaBar]
extrapolation beyond cut with Fermi-Motion model [Ali/Hiller]

Theory with Cuts in M_{X_s}: (to avoid model-dependent extrapolation in experiment)

for $M_{X_s}^2 \ll M_B^2$ and $q^2 < M_B^2$ final state X_s is jetlike: $p_{X_s}^+ \ll p_{X_s}^-$
($2E_{X_s} = p_{X_s}^+ + p_{X_s}^-$ and $M_{X_s}^2 = p_{X_s}^+ p_{X_s}^-$)

$\Rightarrow$ shape function region

at LO in SCET same universal jet and shape functions as in $\bar{B} \to X_s\gamma$ and $\bar{B} \to X_u\ell\nu$

- $M_{X_s}^{\text{cut}} = 1.8\text{GeV}$: $\bar{B}(\bar{B} \to X_s\ell^+\ell^-) = (1.20 \pm 0.15) \times 10^{-6}$
- $M_{X_s}^{\text{cut}} = 2.0\text{GeV}$: $\bar{B}(\bar{B} \to X_s\ell^+\ell^-) = (1.48 \pm 0.14) \times 10^{-6}$

[Lee/Stewart], [Lee/Ligeti/Stewart/Tackmann]

Summary

Included and investigated:

- NNLO QCD and NLO QED corrections in $\Delta B = 1$ effective theory
- almost complete NNLO QCD matrix elements
- NLO QED matrix elements (including collinear log's)
- power corrections $\mathcal{O}[(\Lambda_{\rm QCD}/m_b)^2]$ and also $\mathcal{O}[(\Lambda_{\rm QCD}/m_c)^2]$
- effects due to cut in M_{X_s}

Missing investigations of uncertainties due to

- $\mathcal{O}[(\Lambda_{\rm QCD}/m_b)^3]$ power corrections
- α_s corrections to order $\mathcal{O}[(\Lambda_{\rm QCD}/m_b)^2]$ power corrections

Additional interesting observable: Forward-Backward asymmetry of lepton-pair