Determination of $|V_{ub}|$
from
$\bar{B} \to X_c \ell \bar{\nu}_\ell$ and $\bar{B} \to X_u \ell \bar{\nu}_\ell$

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Heike Boos

Theoretische Physik I, Universität Siegen

in collaboration with:

Thorsten Feldmann, Thomas Mannel, Ben D. Pecjak

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Motivation

$|V_{ub}|$ from $\bar{B} \to X_u \ell \bar{\nu}_\ell$: large background from $\bar{B} \to X_c \ell \bar{\nu}_\ell$

$\Rightarrow$ cuts in phase space: $m_X^2 \sim \Lambda\, m_b$

$\Rightarrow$ decay rate sensitive to (non-perturbative) shape function

$\bar{B} \to X_c \ell \bar{\nu}_\ell$: use similar cuts in phase space

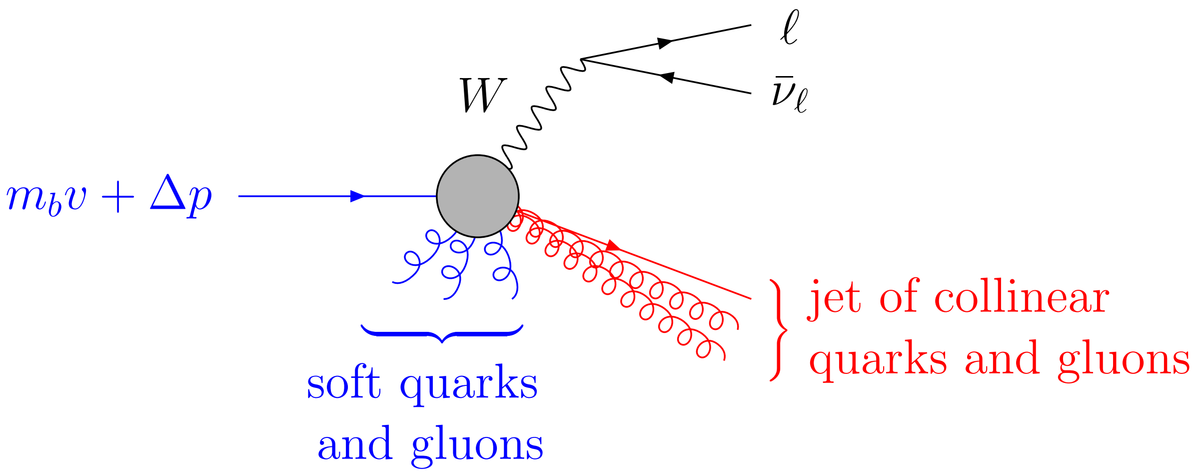
particular power counting $m_c \sim \sqrt{\Lambda\, m_b}$ $\quad$ $(1,5 \text{ GeV} \approx \sqrt{0,5 \cdot 4,8} \text{ GeV})$

$\Rightarrow$ decay rate sensitive to shape function

shape-function independent relation
between $\bar{B} \to X_c \ell \bar{\nu}_\ell$ and $\bar{B} \to X_u \ell \bar{\nu}_\ell$ decay spectra
$\Rightarrow$ extraction of $|V_{ub}|$

$\Rightarrow$ calculate radiative corrections ($\mathcal{O}(\alpha_s)$) to $\bar{B} \to X_c \ell \bar{\nu}_\ell$ in SCET

$\bar{B} \to X_c \ell \bar{\nu}_\ell$ in the shape-function region



- massive collinear modes:

$$E_{jet} = \frac{m_b}{2} + \mathcal{O}(\Lambda) = \mathcal{O}(m_b)$$
$$|p_{jet}|^2 - m_c^2 = \mathcal{O}(\Lambda\, m_b)$$
$$m_c^2 = \mathcal{O}(\Lambda\, m_b)$$

- soft modes: $\Delta p \sim \mathcal{O}(\Lambda)$, $\quad p_s \sim \mathcal{O}(\Lambda)$

Effective field theories for $\bar{B} \to X_c \ell \bar{\nu}_\ell$

QCD

all quark and gluon modes propagate

$\mu_1 \sim m_b$

HQET $\otimes$ SCET

soft and collinear modes for

light quarks and gluons

and massive charm quarks ($m_c \sim \sqrt{\Lambda\, m_b}$)

$\mu_2 \sim \sqrt{\Lambda\, m_b}$

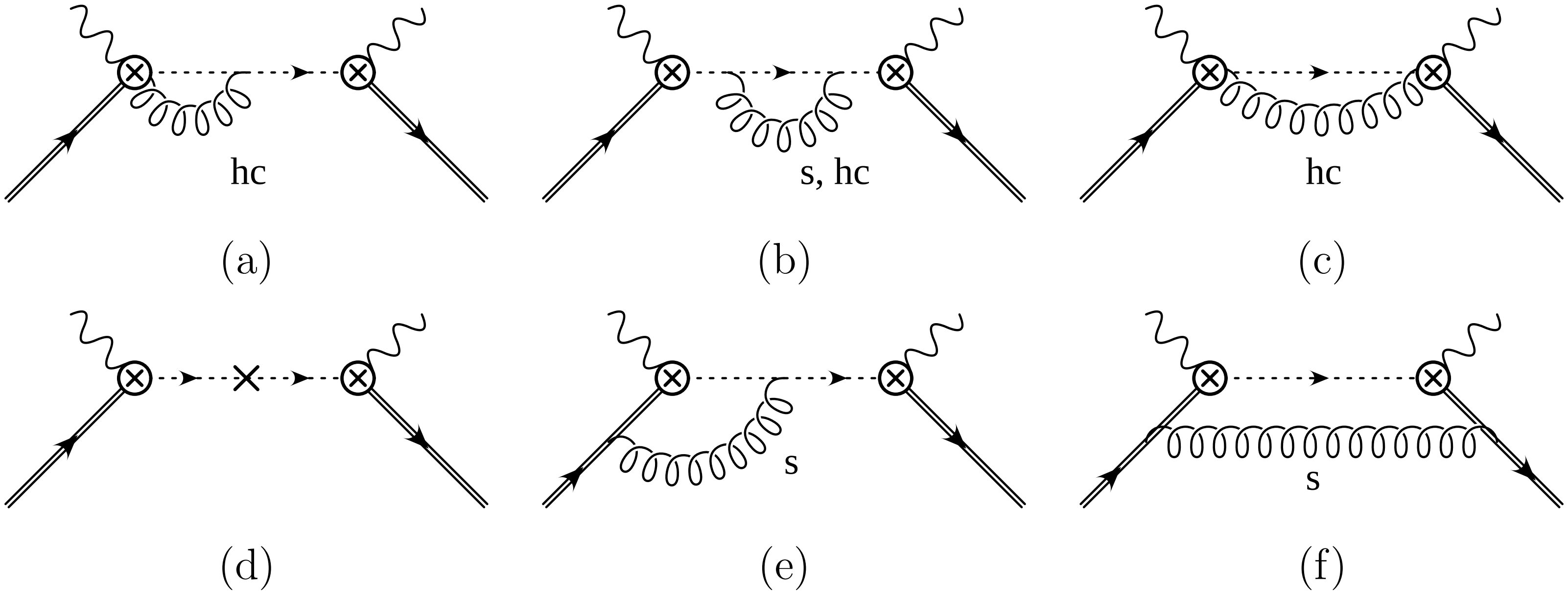
HQET

only soft modes

$\mu_3 \sim \Lambda$

Factorization : $d\Gamma \sim H \cdot J \otimes S$

/// $\bar{B} \to X_c \ell \bar{\nu}_\ell$ in SCET at one loop ///



collinear gluon-exchange + mass counterterm $\Rightarrow J$

H, S : the same as in $\bar{B} \to X_u \ell \bar{\nu}_\ell$ Bosch, Lange, Neubert, Paz

Factorization $d\Gamma \sim H \cdot J \otimes S$ to one-loop α_s-accuracy $\surd$

/// The partially integrated U spectrum ///

consider particular kinematic variable (U is mass-scheme dependent)

$$U = n_- P - \frac{m_c^2}{n_+ P} \qquad \longleftrightarrow \qquad P_+ = n_- P \quad (b \to u)$$

partially integrated spectrum: (cut-off parameter $\Delta \approx 600$ MeV)

$$F_c(\Delta) = \frac{1}{\Gamma_c} \int_0^\Delta dU \, \frac{d\Gamma_c}{dU} = \frac{\Gamma_c(U < \Delta)}{\Gamma_c} = F_u(\Delta) + F_m(\Delta)$$

theoretical result contains shape function $\hat{S}$:

$$F_u(\Delta) \sim \int_0^\Delta d\hat{\omega} \, \hat{S}(\hat{\omega}, \mu_i) \, f_u \left(\frac{m_b(\Delta - \hat{\omega})}{\mu_i^2} \right)$$

$$F_m(\Delta) \sim \int_0^\Delta d\hat{\omega} \, \hat{S}(\hat{\omega}, \mu_i) \, f_m \left(\frac{m_b(\Delta - \hat{\omega})}{m_c^2} \right)$$

Relating $b \longrightarrow c$ and $b \longrightarrow u$ decays

shape-function independent relation $\Longrightarrow$ construct weight function

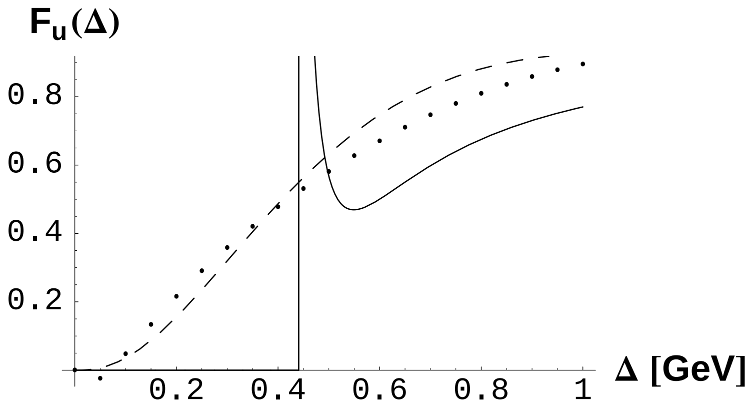
$$F_u(\Delta) = \int_0^\Delta dP_+ \underbrace{\frac{d\Gamma_u}{dP_+}}_{\text{exp.}} = \frac{\Gamma_u}{\Gamma_c} \int_0^\Delta dU\, W(\Delta, U)\, \frac{d\Gamma_c}{dU}$$

$$\simeq \frac{|V_{ub}|^2}{|V_{cb}|^2} \int_0^\Delta dU\, \underbrace{W(\Delta, U)}_{\text{theory}}\, \underbrace{\frac{d\Gamma_c}{dU}}_{\text{exp.}} \qquad (1)$$

$$W(\Delta, U) = 1 - f_m\left(\frac{m_b(\Delta - U)}{m_c^2}\right) + \mathcal{O}(\alpha_s^2) + \mathcal{O}\left(\frac{\Lambda}{m_b}\right)$$

$\left\{ \begin{array}{c} \frac{d\Gamma_c}{dU} \\ W \end{array} \right\}$ mass-scheme dependent $\longleftrightarrow$ (1) mass-scheme independent

/// Illustrate use of weight function ///



$\mu = 1.5$ GeV

- $\left\{ \begin{array}{l} \text{dashed line: theoretical result for } F_u(\Delta) \\ \text{dotted line: } F_u(\Delta) \text{ from weight function and theoretical } U \text{ spectrum} \end{array} \right\}$
- perturbative uncertainties in the weight-function analysis still $15 - 30\%$ (scale dependence)
- illustration of potential resonance effects:
  solid line: $F_u(\Delta)$ from weight function and toy model for U spectrum

Conclusions

- $\bar{B} \to X_c \ell \bar{\nu}_\ell$ in SCET with massive quarks ($m_c \sim \sqrt{\Lambda m_b}$)

- Factorization: $d\Gamma \sim H \cdot J \otimes S$
  - Factorization to one-loop α_s-accuracy, leading order in $\frac{\Lambda}{m_b}$ $\surd$
  - calculation of J to $\mathcal{O}(\alpha_s)$, leading order in $\frac{\Lambda}{m_b}$

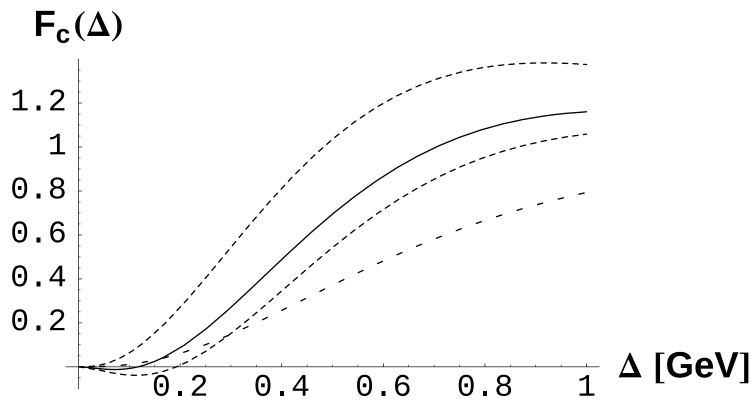
- particular kinematic variable

  $U = n_- P - \frac{m_c^2}{n_+ P} \qquad \longleftrightarrow \qquad P_+ = n_- P \ \ (b \to u)$

shape-function independent relation between
partially integrated $\bar{B} \to X_c \ell \bar{\nu}_\ell$ and $\bar{B} \to X_u \ell \bar{\nu}_\ell$ decay spectra
$\Rightarrow$ determination and crosscheck of $|V_{ub}/V_{cb}|$

- wait for experimental information on U spectrum in $\bar{B} \to X_c \ell \bar{\nu}_\ell$

///Numerical predictions for partially integrated U spectrum///



NLO Prediction
solid line: PS scheme
upper dotted line: pole scheme
lower dotted line: $\overline{\text{MS}}$ scheme

dashed line: LO result

- $\Delta \sim 600$ MeV: NLO corrections are large and positive
- Above some critical value Δ_{max}:
  $F_c > 1 \Rightarrow$ result should not be trusted anymore
- $\Delta_{\text{max}} \approx 480$ MeV (pole scheme), $\Delta_{\text{max}} \approx 700$ MeV (PS scheme),
  $\Delta_{\text{max}} \approx 860$ MeV ($\overline{\text{MS}}$ scheme)

keep in mind: U mass-scheme dependent