Higher orders in $B \longrightarrow \pi\pi$ in QCD factorization and SCET

Sebastian Jäger

Institut für Theoretische Physik E, RWTH Aachen

in collaboration with Martin Beneke

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$B \to \pi\pi$ amplitudes

Model-independent parameterization (assuming isospin,neglecting EW penguins):

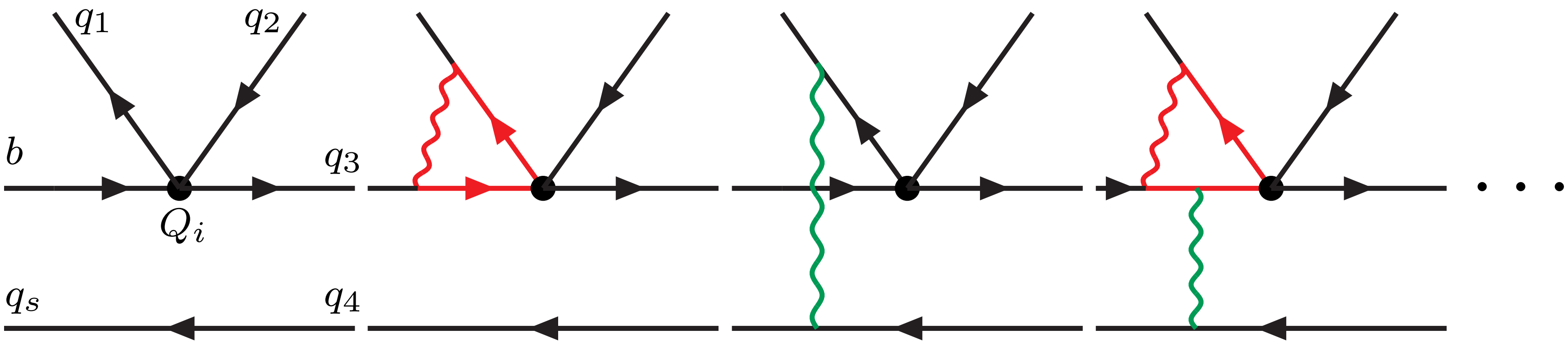
$$\begin{aligned}\sqrt{2}\mathcal{A}(B^- \to \pi^-\pi^0) &= e^{-i\gamma}(\hat{T}+\hat{C})\\ \mathcal{A}(\bar{B}^0 \to \pi^+\pi^-) &= e^{-i\gamma}\hat{T}-\hat{P}\\ \sqrt{2}\mathcal{A}(\bar{B}^0 \to \pi^0\pi^0) &= e^{-i\gamma}\hat{C}+\hat{P}\end{aligned}$$

$\hat{T}, \hat{C}, \hat{P}$: physical, including two strong phases.

$$\mathcal{A}(\bar{B} \to \pi\pi) = \overbrace{C_i(\mu)}^{\text{Wilson coefficient}} \quad \underbrace{\langle \pi\pi | \mathcal{O}_i(\mu) | \bar{B} \rangle}_{\text{hadronic matrix element}}$$

- $C_i(\mu)$ contains effects of hard gluons $(k^2 > \mu^2 \sim m_b^2)$ and heavy particles (W, Z, t, new physics) — find from expanding Feynman diagrams in p/M_W (external momentum/heavy mass)
- $\langle \pi\pi | \mathcal{O}_i(\mu) | \bar{B} \rangle$ contain all dynamics below the scale μ

QCD corrections: Scales m_b and below



- b-quark and spectator in B-meson are soft, quarks representing energetic light mesons collinear:

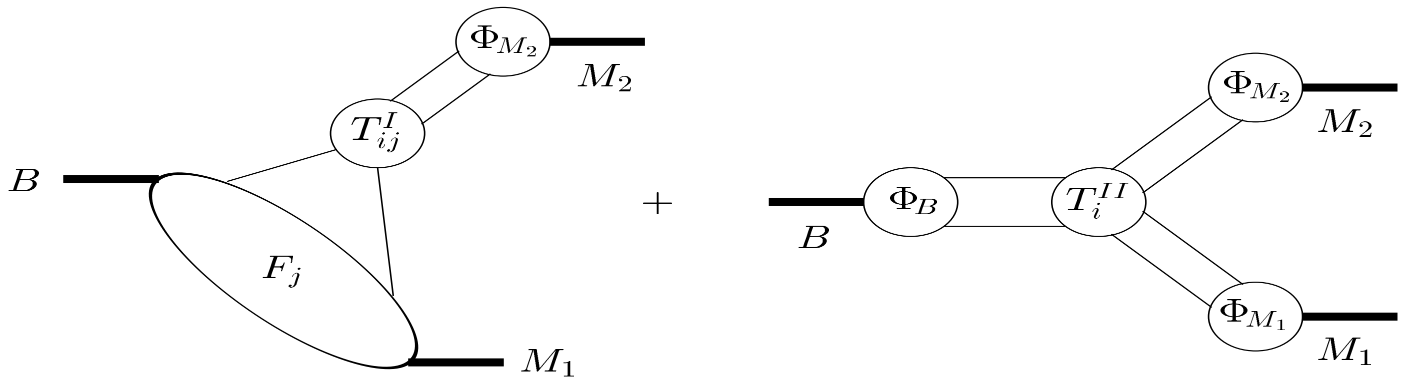
$$
\begin{aligned}
p_b &= m_b(n_+ + n_-)/2 + \mathcal{O}(\Lambda_{\rm QCD}), \qquad & n_+^2 = n_-^2 = 0 \\
p_{q_s} &= \mathcal{O}(\Lambda_{\rm QCD}) & \\
p_{q_{1,2}} &= \overset{(-)}{u}\; m_b n_+/2 + \mathcal{O}(\Lambda_{\rm QCD}), & u + \bar{u} = 1 \\
p_{q_{3,4}} &= \overset{(-)}{v}\; m_b n_-/2 + \mathcal{O}(\Lambda_{\rm QCD}), & v + \bar{v} = 1
\end{aligned}
$$

- Can form invariants $p_b \cdot p_{q_i} \sim m_b^2$, $p_{q_s} \cdot p_{q_4} \sim \Lambda m_b \equiv \lambda^2$, suggests expanding amplitudes in λ/m_b

Factorization formula for hadronic B-decays

$$\langle M_1 M_2|\mathcal{O}_i|B\rangle = \overbrace{F^{B\to M_1}}^{\text{form factor}} \int du\, T_i^{\mathrm{I}}(u)\phi_{M_2}(u) + \int du\, dv\, d\omega T_i^{\mathrm{II}} \overbrace{\phi_B(\omega)\phi_{M_1}(v)\phi_{M_2}(u)}^{\text{light-cone distribution amplitudes}}$$

Beneke, Buchalla, Neubert, Sachrajda 1999,2001



Soft-collinear factorization: $T^{\mathrm{II}}(u,v,\omega) = \int dv' H^{\mathrm{II}}(u,v')J(v',\omega) \equiv H^{\mathrm{II}} \star J$

$$\left.\begin{aligned} T^{\mathrm{I}} &= 1 + T^{\mathrm{I}(1)}\alpha_s(m_b) + \ldots \\ H^{\mathrm{II}} &= 1 + H^{\mathrm{II}(1)}\alpha_s(m_b) + \ldots \\ J &= J^{(1)}\alpha_s(\sqrt{\Lambda m_b}) + J^{(2)}\alpha_s(\sqrt{\Lambda m_b})^2 + \ldots \end{aligned}\right\} \text{perturbative}$$

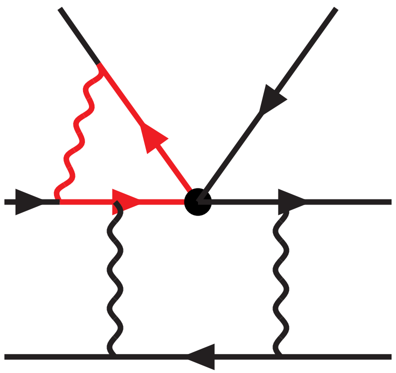
T^{I}, T^{II}: process dependence and strong phases

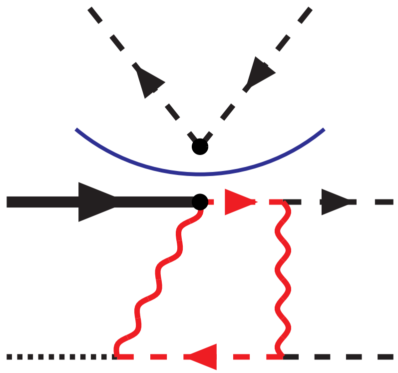
Soft-collinear effective theory

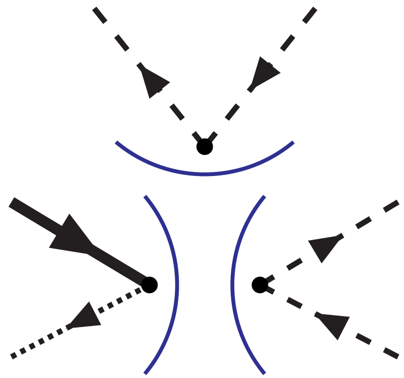
T^{I}, H^{II}, J are Wilson coefficients in $\mathrm{SCET_I}$ and $\mathrm{SCET_{II}}$.

Bauer, Fleming, Luke 2001; Bauer, Fleming, Pirjol, Stewart 2001,2003;
Beneke, Chapovsky, Diehl, Feldmann 2002

full QCD	remove virtualities $\mathcal{O}(m_b^2)$ $\longrightarrow$	$\mathrm{SCET_I}$	remove virtualities $\mathcal{O}(\Lambda m_b)$ $\longrightarrow$	$\mathrm{SCET_{II}}$
all modes	$\downarrow$ $T^{\mathrm{I}}, H^{\mathrm{II}}$	hard-collinear,collinear, soft modes	$\downarrow$ J	collinear,soft modes







Transition between the scales by RGE - resums large logarithms

QCD factorization in SCET$_{\mathrm{I}}$

To match diagrams without/with spectator interactions, need

$$O^{\mathrm{I}} = \left(\bar\chi(tn_-)\frac{\not n_-}{2}\gamma_5\chi(0)\right)\overbrace{\left(\bar\xi(sn_+)\not n_+ h_v(0)\right)}^{\text{A-type current}} \equiv Q(t)J^A(s)$$

$$O^{\mathrm{II}} = \frac{1}{m_b}\left(\bar\chi(tn_-)\frac{\not n_-}{2}\gamma_5\chi(0)\right)\underbrace{\left(\bar\xi(s_1n_+)\not n_+ i\not D_{\perp c}(s_2n_+)h_v(0)\right)}_{\text{B-type current}} \equiv Q(t)J^B(s_1,s_2)$$

- $\not n_-\xi = \not n_+\chi = 0$ ((hard-)collinear quarks). $iD_{\perp c} = i\partial_\perp + gA_{\perp c}$ (transverse (hard-)collinear gluon). h_v HQET heavy quark field
- Nonlocal along n_+,n_-
- amplitudes factorize up to $\mathcal{O}(1/m_b)$ (not $1/\sqrt{\Lambda m_b}$) corrections
- $[\mathrm{F.T.}\langle\pi|Q(t)|0\rangle](u) = \phi_\pi(u)$ defines π light-cone distribution amplitude

QCD factorization in SCET$_{\mathrm{I}}$ (2)

$$\mathcal{O}_i \to \mathcal{H}_{\mathrm{SCET_I}} = \int du\, C^{\mathrm{I}}(u) Q(u) J_A + \int du\, dv'\, C^{\mathrm{II}}(u,v') Q(u) J_B(v')$$

This suggests (cf Bauer,Pirjol,Rothstein,Stewart 2004)

$$\langle \pi\pi|\mathcal{O}_i|\bar{B}\rangle = C^{\mathrm{I}} * \phi_\pi \underbrace{\langle \pi|J_A|\bar{B}\rangle}_{\xi_\pi \text{(soft form factor)}} + C^{\mathrm{II}} * \phi_\pi * \underbrace{\langle \pi|J_B(v')|\bar{B}\rangle}_{\Xi(v')}$$

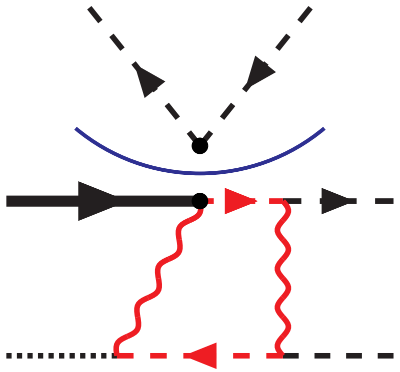
but $$F^{B\to\pi} = C_A \xi_\pi + C_B * \underbrace{\Xi(v')}_{\text{same as above}}$$ Beneke,Feldmann 2000,2003

redefine $Q(u)J_A \longrightarrow Q(u)(C_A J_A + C_B * J_B)$, $C^{\mathrm{I}} \to T^{\mathrm{I}}$, $C^{\mathrm{II}} \to H^{\mathrm{II}}$

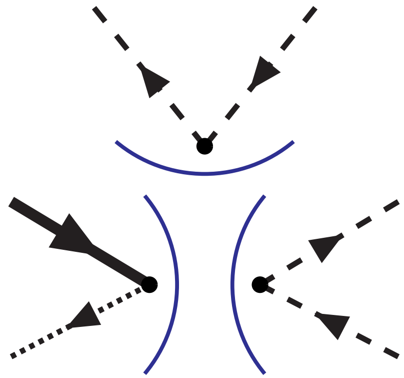
$$\Rightarrow \quad \langle \pi\pi|\mathcal{O}_i|\bar{B}\rangle = \overbrace{T^{\mathrm{I}}}^{\text{BBNS 1999}} * \phi_\pi\, F^{B\to\pi} + \overbrace{H^{\mathrm{II}}}^{\text{NLO is our goal}} * \underbrace{\Xi}_{\to\text{below}}$$

$T^{\rm II} = H^{\rm II} * J$

Second matching step onto $\mathrm{SCET_{II}}$ is identical to form factor case



$\Rightarrow$



$$\int dv' H^{\rm II}(u,v')Q(u)J_B(v') \quad \Rightarrow \quad \int dv \int d\omega \underbrace{\int dv' H^{\rm II}(u,v') J(v',v,\omega)}_{T^{\rm II}(u,v)} Q(u)\tilde{Q}(v)P(\omega)$$

$\langle \bar{B}|P(\omega)|0\rangle = \phi_{B+}(\omega)$ defines B-meson LCDA

$$\Rightarrow H^{\rm II} * \Xi = T^{\rm II} * \phi_\pi * \phi_\pi * \phi_{B+}$$

Jet function known to $\mathcal{O}(\alpha_s^2)$

Hill, Becher, Lee, Neubert 2004; Beneke, Yang 2005; Kirillin 2005

Matching onto SCET$_{\rm I}$- current-current diagrams

- Partonic QCD amplitude $\stackrel{!}{=}$ partonic matrix element

$$\langle \bar{q}qqg|\mathcal{O}^u_{1,2}|b\rangle_{\rm QCD} = T^{\rm I} * \langle \bar{q}q|Q(u)|0\rangle\langle qg|J|b\rangle + H^{\rm II} * \langle \bar{q}q|Q(u)|0\rangle * \langle qg|J_B(v')|b\rangle$$

- (IR) divergence structure restricted on the r.h.s, but (a priori) not on l.h.s.
- LHS: compute about 50 diagrams in QCD, with dimensional regularization, as function of large momentum components
- RHS: compute one-loop matrix elements, main issue: evanescent operators
  - $D_2 = \frac{\not{n}_-}{2}\gamma^{\alpha\perp} \otimes \frac{\not{n}_+}{2}\gamma_{\mu\perp}\gamma_{\alpha\perp}$ vanishes in $d=4$, but appears at tree level
  - must ensure (by finite renormalization) that $\langle D_2\rangle^{(1-{\rm loop})} = 0$
- read off $H^{\rm II(1)}$ from equation

Analytical results

$$H^{\mathrm{II}(1)} \propto r_{1,2}$$

$$\begin{aligned}
r_1 = C_F \Big[& -\frac{1}{2\bar u}\ln^2\frac{m_b^2}{\mu^2} + \Big(6 - \frac{5}{2\bar u} + \frac{2}{\bar u}\ln\bar u\Big)\ln\frac{m_b^2}{\mu^2} + \frac{u}{\bar u}\big[V(u)+18\big] \\
& - \frac{2u}{\bar u}F(v,u) + \frac{2u^3}{(\bar v-u)^3}F(v,\bar u) - \frac{2}{\bar u}\Big(\ln\bar u + \ln\bar v\Big)i\pi - \frac{1}{\bar u}\Big(9 + \frac{5}{12}\pi^2\Big) \\
& + \Big(\frac{u}{\bar v-u} - \frac{2u^2}{(\bar v-u)^2} - \frac{2(3u-2)}{\bar u}\Big)\big[\log u - i\pi\big] - \frac{2(1-uv)}{u\bar u v\bar v}\ln(1-uv) \\
& - \Big(\frac{u(2-4u+u^2)}{\bar u^2(\bar v-u)} - \frac{2(2-u)u^2}{\bar u(\bar v-u)^2} + \frac{\bar u - v + 4u\bar u v}{u\bar u^2 v^2}\Big)\ln(1-\bar u v) \\
& + \frac{1}{\bar u}(\ln^2\bar u - \ln^2\bar v) + \Big(\frac{u}{\bar v-u} - \frac{2u^2}{(\bar v-u)^2} + \frac{\bar v + 3uv^2}{uv^2}\Big)\ln\bar v \\
& + \Big(3 + \frac{2}{u\bar v} - 2\ln v + \Big(2 + \frac{2}{\bar u}\Big)\ln\bar v\Big)\ln\bar u + \Big(3 + \frac{2}{\bar u}\ln\bar v\Big)\ln v \\
& + \Big(\frac{1-3u\bar u}{\bar u^2} + \frac{2(3u-2)}{\bar u} + 2\ln v + \frac{2u}{\bar u}\ln\bar v\Big)\ln u + \frac{2}{\bar u}\mathrm{Li}_2(\bar v)\Big] \\
+ \Big(C_F - \frac{C_A}{2}\Big)\Big[& -\frac{2}{\bar u\bar v}\ln v\,\ln\frac{m_b^2}{\mu^2} + \frac{1}{\bar u}V(u) + \frac{2u}{v-u}F(v,u) + \frac{2u^2}{(\bar v-u)^2}F(v,\bar u) \\
& + 2i\pi\Big(\frac{\bar v}{\bar v-u} + \frac{u}{\bar u}\ln\frac{u}{\bar u} + \frac{v}{\bar u\bar v}\ln v + \frac{1}{\bar u}\ln\bar v\Big) + \frac{1}{\bar u}\Big[\ln^2\bar v + \frac{\pi^2}{3}\Big] + \frac{u}{\bar u}(\ln^2 u - \ln^2\bar u) \\
& - \frac{1+\bar v}{\bar u\bar v}\ln^2 v + \Big(\frac{2-3u}{u\bar u} + \frac{2}{u\bar v}\Big)\ln\bar u - \frac{2(1-uv)}{u\bar u v\bar v}\ln(1-uv) \\
& + \Big(\frac{2(1-3u^2+u^3)}{u\bar u v} + \frac{2(1-\bar v^2\bar u)}{\bar u v\bar v} + \frac{2(2-u)u}{v(\bar v-u)}\Big)\ln(1-\bar u v) \\
& + \Big(\frac{1+2u\bar u}{\bar u^2} - \frac{2(1-uv)}{\bar u^2\bar v} + \frac{2u(u\bar u - v)}{\bar u^2(\bar v-u)}\Big)\ln u \\
& + \Big(\frac{3(1+\bar v)}{\bar u\bar v} + \frac{2}{\bar u}\ln u - \frac{2}{\bar u\bar v}\ln\bar u\Big)\ln v \\
& - \Big(\frac{2-5u-4u^2+2u^3}{u\bar u v} + \frac{(1+2u)\bar v}{\bar u v} + \frac{2u\bar u}{v(\bar v-u)} + \frac{2u}{\bar u}\ln u \\
& \qquad + 2\ln\bar u + \frac{2(1-2u)}{u\bar u\bar v}\ln v\Big)\ln\bar v \\
& + 2\Big\{-\frac{1+2u\bar v}{\bar u\bar v}\mathrm{Li}_2(u) + \frac{1-u\bar v+2u^2\bar v}{u\bar u\bar v}\mathrm{Li}_2(\bar u) + \frac{1+u\bar v}{\bar u\bar v}\mathrm{Li}_2(uv) \\
& \qquad - \frac{1-u\bar u\bar v}{u\bar u\bar v}\mathrm{Li}_2(\bar u v) - \frac{1-2u+u\bar v}{u\bar u\bar v}\mathrm{Li}_2(\bar v)\Big\}\Big],
\end{aligned}$$

$$\begin{aligned}
r_2 = {} & \frac{1}{2\bar u}\big[V(u)+2\big] + \frac{u\bar v}{\bar u(v-u)}F(v,u) + \frac{u^2\bar v}{(\bar v-u)^3}F(v,\bar u) + \frac{\bar v(\bar v-3u)}{2(\bar v-u)^2}\big[\ln\bar v - i\pi\big] \\
& + \Big(\frac{1+u^2}{2\bar u^2} + \frac{u}{2\bar u^2\bar v} - \frac{u^2}{(\bar v-u)^2} - \frac{u}{2(\bar v-u)}\Big)\ln u + \frac{\bar u+\bar v}{u\bar u\bar v}\ln\bar u \\
& + \Big(\frac{3}{2\bar u} - \frac{1}{2\bar u\bar v} + \frac{\ln u}{\bar u} - \frac{\ln\bar u}{\bar u}\Big)\ln v - \Big(\frac{1}{u\bar u v} + \frac{1}{u\bar v}\Big)\ln(1-uv) \\
& + \Big(-\frac{1}{2} - \frac{u}{2\bar u^2\bar v} + \frac{u^2(\bar u^2+v)}{\bar u^2(\bar v-u)^2} + \frac{u(2+u^2)}{2\bar u^2(\bar v-u)}\Big)\ln(1-\bar u v),
\end{aligned}$$

where

$$\begin{aligned}
F(v,w) &= 2\,\mathrm{Li}_2\Big(-\frac{\bar v w}{\bar w}\Big) + 2\,\mathrm{Li}_2(w) - \mathrm{Li}_2(vw) + \frac{1}{2}\ln^2\frac{\bar w}{\bar v} + i\pi\ln\frac{\bar w}{\bar v}, \\
V(u) &= 6\ln\frac{m_b^2}{\mu^2} - 18 + 3\Big(\frac{1-2u}{\bar u}\ln u - i\pi\Big) \\
& \quad + \Big\{2\,\mathrm{Li}_2(u) - \ln^2 u + \frac{2\ln u}{\bar u} - (3+2i\pi)\ln u - (u\leftrightarrow\bar u)\Big\}.
\end{aligned}$$

Tree amplitudes in QCDF/SCET

- Two ways to insert $\mathcal{O}_1^u, \mathcal{O}_2^u$: $O^{\rm II} \to O_1^{\rm II}$, $O_2^{\rm II}$
- Their matrix elements (convolutions) define two “topological” amplitudes α_1, α_2, the generalizations of the color-allowed and color-suppressed tree amplitudes.

$$\begin{aligned}
\sqrt{2}\mathcal{A}(B^- \to \pi^-\pi^0) &= V_{ub}V_{ud}^* A_{\pi\pi}(\alpha_1 + \alpha_2) + \text{annihilation} \\
\mathcal{A}(\bar{B}^0 \to \pi^+\pi^-) &= V_{ub}V_{ud}^* A_{\pi\pi}\alpha_1 + \text{QCD penguins + annihilation} \\
-\mathcal{A}(\bar{B}^0 \to \pi^0\pi^0) &= V_{ub}V_{ud}^* A_{\pi\pi}\alpha_2 + \text{QCD penguins + annihilation}
\end{aligned}$$

$$A_{\pi\pi} = i\frac{G_F}{\sqrt{2}} m_B^2 f_\pi f_+^{B\pi}(0)$$

$$\alpha_1 = C_1 + 1/N_c C_2 + \mathcal{O}(\alpha_s) C_2 + \mathcal{O}(\alpha_s^2) C_1 + \dots \qquad \text{color-allowed tree}$$

$$\alpha_2 = C_2 + 1/N_c C_1 + \mathcal{O}(\alpha_s) C_1 + \mathcal{O}(\alpha_s^2) C_1 + \dots \qquad \text{color-suppressed tree}$$

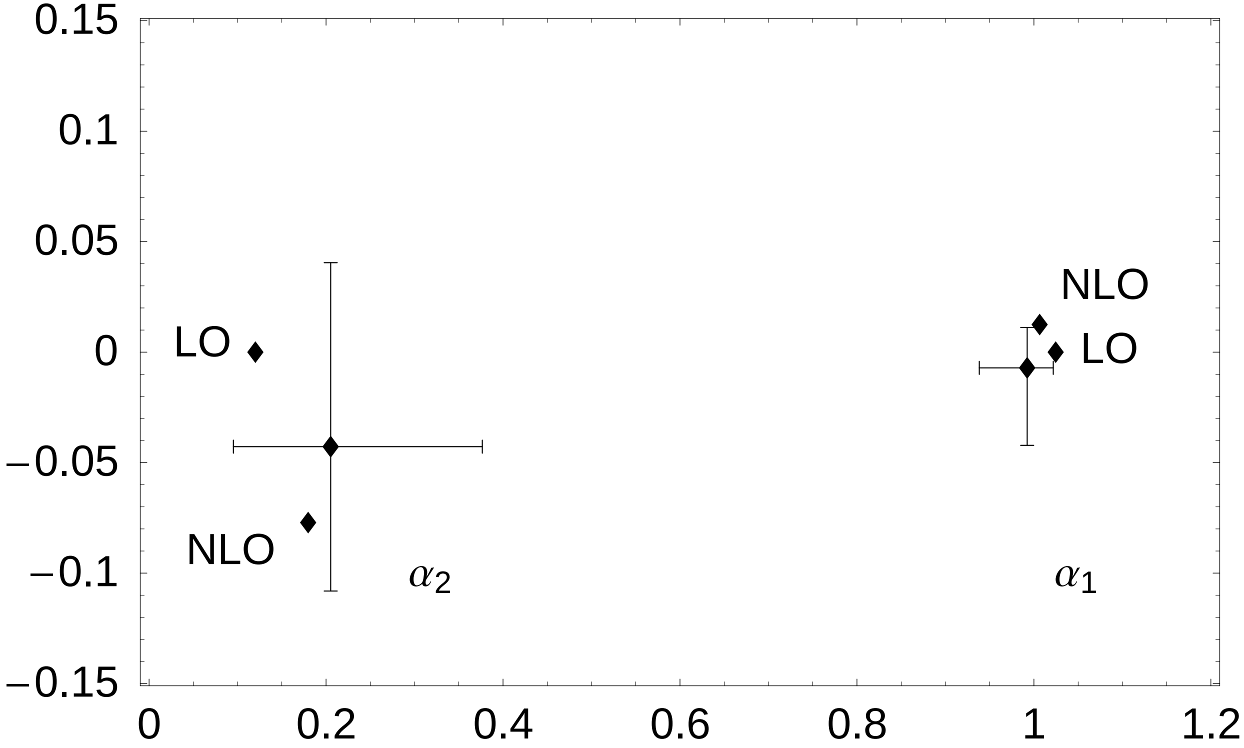
Choice of parameters

Parameter	Value/Range	Parameter	Value/Range
$\Lambda^{\overline{\text{MS}}(5)}$	0.225	μ_b	$4.8^{+4.8}_{-2.4}$
m_c	1.3 ± 0.2	μ_{hc}	1.5 ± 0.6
m_s(2 GeV)	0.09 ± 0.02	f_{B_d}	0.20 ± 0.03
m_b	4.8	$f_+^{B\pi}(0)$	0.28 ± 0.05
$\bar{m}_b(\bar{m}_b)$	4.2	λ_B(1 GeV)	0.35 ± 0.15
$\lvert V_{cb}\rvert$	0.0415 ± 0.0010	σ_1(1 GeV)	1.5 ± 1
$\lvert V_{ub}/V_{cb}\rvert$	0.09 ± 0.02	σ_2(1 GeV)	3 ± 2
γ	$(70 \pm 20)^\circ$	a_2^π(2 GeV)	0.1 ± 0.2

- $\lambda_B, \sigma_{1,2}$ parameterize ϕ_{B_+} , a_2^π the deviation of ϕ_π from asymptotic
- μ_b, μ_{hc} are the hard and hard-collinear factorization (renormalization) scales

Findings - α_1 and α_2

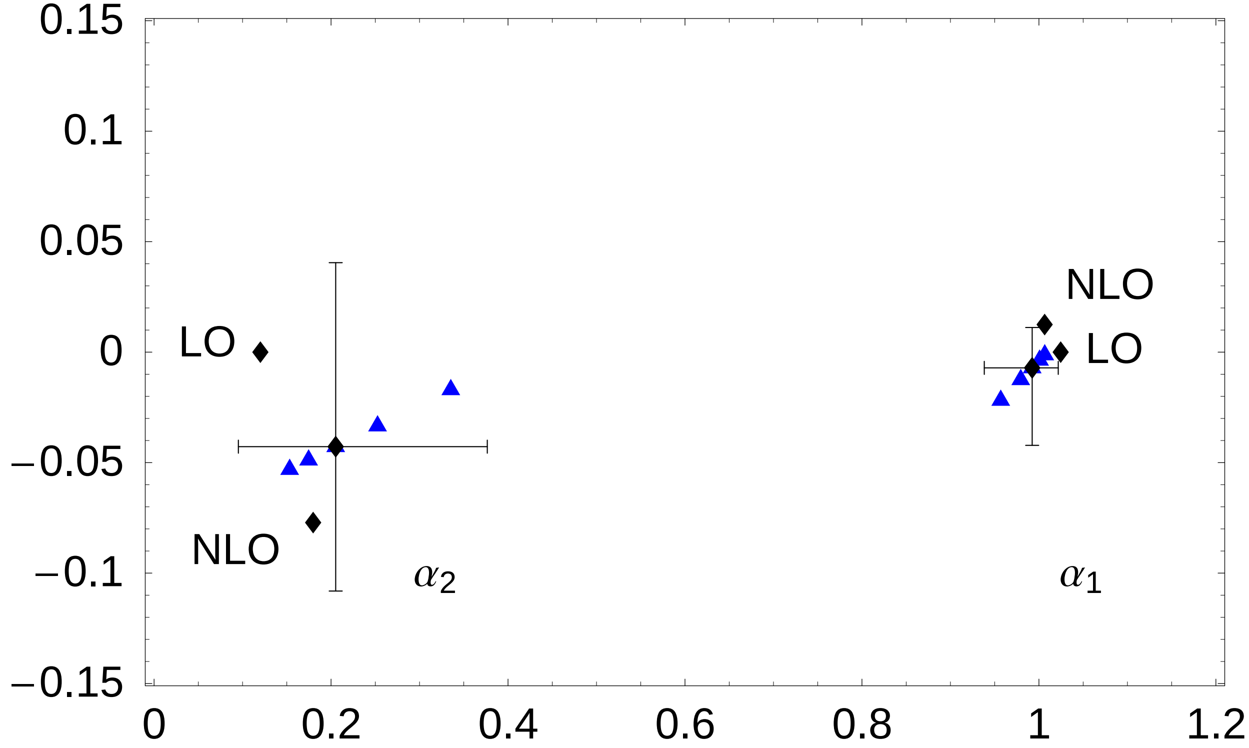
Vary all input parameters and the two factorization scales



- Closeness of LO, NLO, NNLO validates perturbative approach
- For NNLO, errors combined in quadrature; dominant: hard-collinear factorization scale, power corrections

Findings - α_1 and α_2

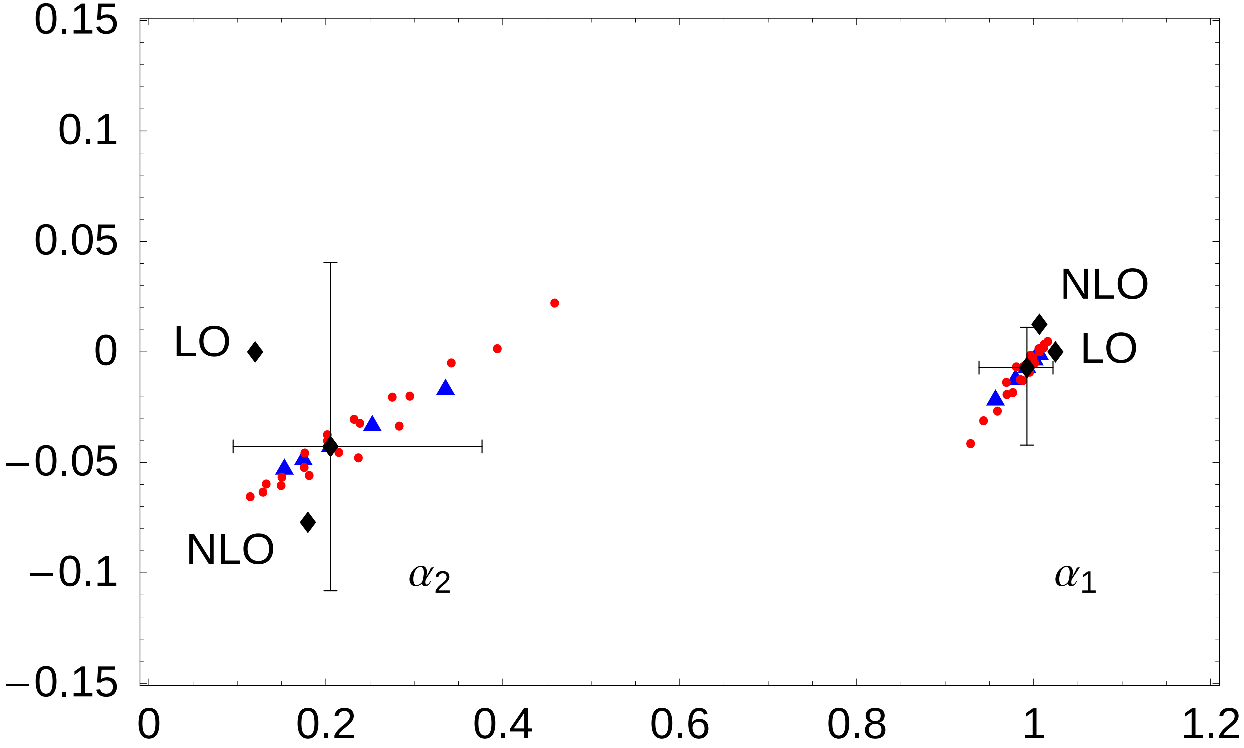
Vary all input parameters and the two factorization scales



Sensitivity to λ_B

Findings - α_1 and α_2

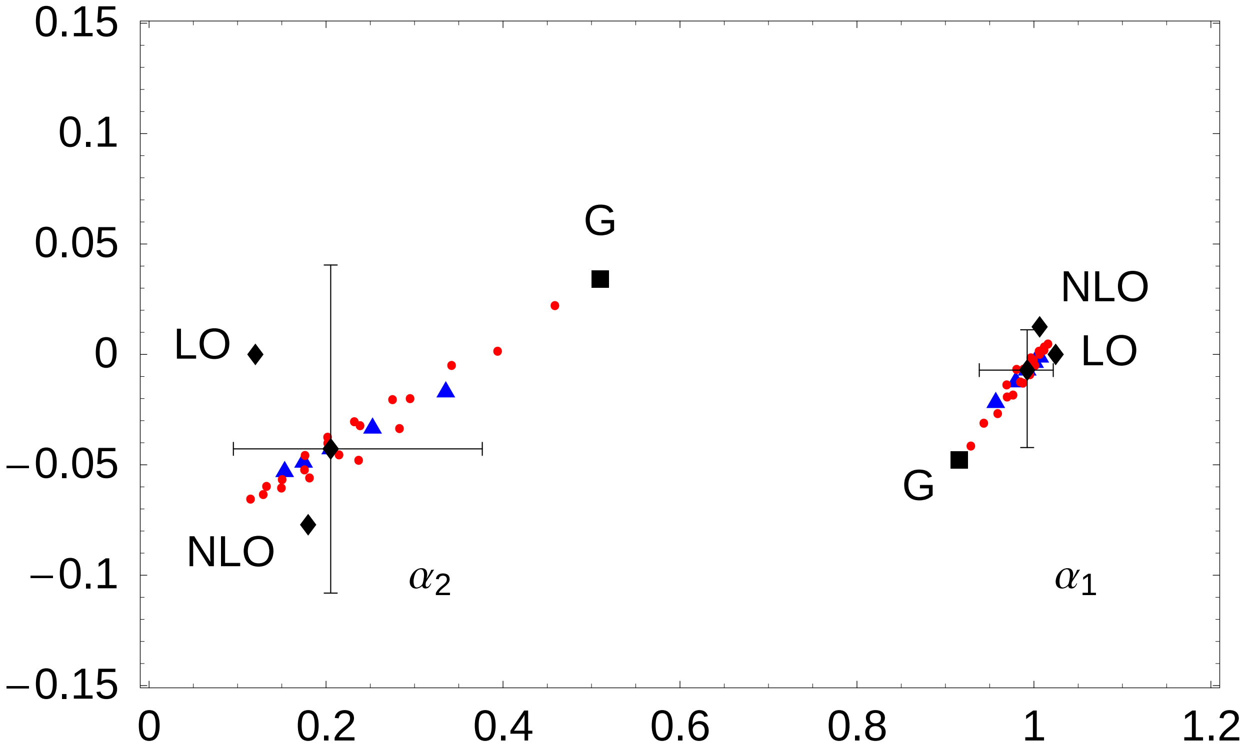
Vary all input parameters and the two factorization scales



Sensitivity to λ_B combines almost linearly with Gegenbauer moment one!

Findings - α_1 and α_2

Vary all input parameters and the two factorization scales



"G" shows result of two-parameter fit of $f_+(0)\lambda_B$ and a_2^π (fit to branching ratios).

Numerical results

$$\alpha_1(\pi\pi) = 0.992^{+0.029}_{-0.054} + (-0.007^{+0.018}_{-0.035})i$$
$$\alpha_2(\pi\pi) = 0.205^{+0.171}_{-0.110} + (-0.043^{+0.083}_{-0.065})i$$

- no large complex phases
- some enhancement of color-suppressed tree possible

For BRs, use fit values for $f_+(0), \lambda_B, a_2^\pi$ (parameter set “G” $\approx$ S4)

$10^6\,\text{Br}(B^- \to \pi^-\pi^0) = 5.5^{+0.3}_{-0.3}(\text{CKM})^{+0.5}_{-0.4}(\text{hadr.})^{+0.9}_{-0.8}(\text{pow.})$ [Exp: 5.5 ± 0.6]

$10^6\,\text{Br}(\bar{B}^0 \to \pi^+\pi^-) = 5.0^{+0.8}_{-0.9}(\text{CKM})^{+0.3}_{-0.5}(\text{hadr.})^{+1.0}_{-0.5}(\text{pow.})$ [Exp: 5.0 ± 0.4]

$10^6\,\text{Br}(\bar{B}^0 \to \pi^0\pi^0) = 0.73^{+0.27}_{-0.24}(\text{CKM})^{+0.52}_{-0.21}(\text{hadr.})^{+0.35}_{-0.25}(\text{pow.})$ [Exp: 1.45 ± 0.29]

- CKM “error” may be interpreted as sensitivity
- Dominant “hadronic” error: hard-collinear factorization scale μ_{hc}

Conclusion and outlook

- First part of NNLO contributions to the QCD factorization formula available. This also constitutes a partial proof of factorization at NNLO.
- Perturbation expansion well behaved also at the hard-collinear scale, scale uncertainties are moderate
- Enhancement of the color-suppressed tree, also with respect to the color-allowed tree, due to 1-loop hard spectator scattering
- Branching fractions, and particularly ratios (and double ratio) in better agreement with experiment. Better determinations of hadronic parameters ($F^{B\to\pi}$,a_2^π,λ_B) desirable
- Penguin contributions important for CP asymmetries and many branching fractions, but not yet available at NNLO
- Full NNLO formula also requires two-loop vertex corrections.