Azimuthal asymmetries in polarized top quark decays at $O(\alpha_s)$

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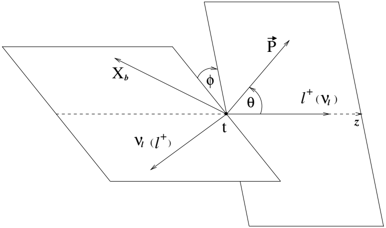
Outline

The general angular decay distribution

The decay $t(\uparrow) \rightarrow b + W^+(\ell^+\nu)$:

$$\frac{d\Gamma}{dx_{\ell,\nu}\, d\cos\theta\, d\phi} = \frac{1}{4\pi}\left[\frac{d\Gamma_A}{dx_{\ell,\nu}} + P\left(\frac{d\Gamma_B}{dx_{\ell,\nu}}\cos\theta + \frac{d\Gamma_C}{dx_{\ell,\nu}}\sin\theta\cos\phi\right)\right]$$



$$\vec{P} = (\sin\theta\cos\phi,\ \sin\theta\sin\phi,\ \cos\theta)$$

$$x_{\ell,\nu} = \frac{2E_{\ell,\nu}}{m_t}$$

- A and B results agree with Kühn, Jezabek and Czarnecki.
- C result is new.

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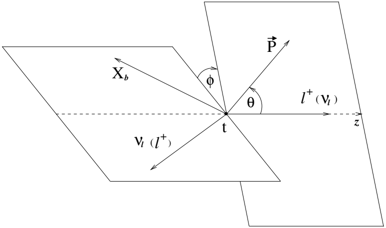

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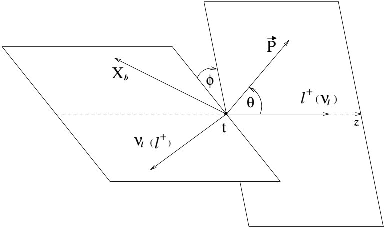

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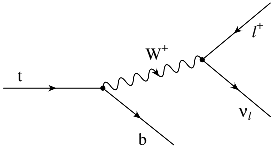
Born term results



The matrix element : $M \to i \frac{G_F V_{tb}}{\sqrt{2}} H_\mu L^\mu$

with leptonic and hadronic currents

$$H_\mu = \bar{u}_b \gamma_\mu (1 - \gamma_5) u_t, \qquad \text{hadronic current,}$$
$$L_\mu = \bar{u}_\nu \gamma_\mu (1 - \gamma_5) v_\ell, \qquad \text{leptonic current.}$$

The decay rate

$$\overline{|M|^2} = \sum_{s_b, s_\ell, s_\nu} |M|^2 = \sum_{s_b, s_\ell, s_\nu} MM^\dagger \to \frac{G_F^2 |V_{tb}|^2}{2} H_{\mu\nu} L^{\mu\nu}$$

with hadron and lepton tensors

$$H_{\mu\nu} = \sum_{s_b} H_\mu H_\nu^\dagger, \qquad L_{\mu\nu} = \sum_{s_\ell, s_\nu} L_\mu L_\nu^\dagger.$$

- $\star$ narrow resonance approximation
- $\star$ zero bottom quark mass

LO differential rate in lepton energy

$$\frac{d\Gamma_A}{dx_\ell} = \frac{d\Gamma_B}{dx_\ell} = \Gamma_F\, 2\pi \frac{m_W}{\Gamma_W} 6x_\ell(1-x_\ell)y; \qquad \frac{d\Gamma_C}{dx_\ell} = 0.$$

(the lepton momentum defines the z–axis)

- $\Gamma_F = \frac{G_F^2 m_t^5}{192\pi^3}|V_{tb}|^2, \;\; y = \frac{m_W^2}{m_t^2}, \;\; x_\ell = \frac{2E_\ell}{m_t}, \;\; x_\nu = \frac{2E_\nu}{m_t}$

LO differential rate in neutrino energy

$$\frac{d\Gamma_A}{dx_\nu} = \Gamma_F\, 2\pi \frac{m_W}{\Gamma_W} 6(x_\nu - y)(1 - x_\nu + y),$$

$$\frac{d\Gamma_B}{dx_\nu} = \Gamma_F\, 2\pi \frac{m_W}{\Gamma_W} 6(x_\nu - y)(1 - x_\nu + y - \frac{2y}{x_\nu}),$$

$$\frac{d\Gamma_C}{dx_\nu} = \Gamma_F\, 2\pi \frac{m_W}{\Gamma_W} 12(x_\nu - y)\sqrt{y(1-x_\nu)(x_\nu - y)}$$

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The LO integrated rate $(y \leq x \leq 1)$

$$\Gamma_A = \Gamma_B = \Gamma_F\, 2\pi \frac{m_W}{\Gamma_W} y(1-y)^2(1+2y), \quad \Gamma_C = 0\,.$$

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\Gamma_A &= \Gamma_F\, 2\pi \frac{m_W}{\Gamma_W} y(1-y)^2(1+2y) \\
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\Gamma_C &= \Gamma_F\, 2\pi \frac{m_W}{\Gamma_W} \frac{3}{2}\pi \sqrt{y}\, \left(1 - 6\,y + 8\,y\sqrt{y} - 3\,y^2\right)\,.
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(the neutrino momentum defines the z–axis)

- *The next step : NLO corrections*

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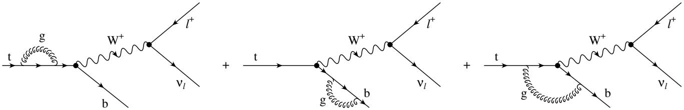
Next–to–leading order corrections

- The virtual corrections
- The real corrections

The virtual corrections



The renormalized hadron current of $O(\alpha_s)$ can be written as:

$$H_\mu^{(1)} = \bar{u}_b(J_\mu^V - J_\mu^A)u_t.$$

The covariant expansion of J_μ^V and J_μ^A is:

$$\begin{aligned} J_\mu^V &= \gamma_\mu F_1^V + p_{t,\mu} F_2^V + p_{b,\mu} F_3^V, \\ J_\mu^A &= (\gamma_\mu F_1^A + p_{t,\mu} F_2^A + p_{b,\mu} F_3^A)\gamma_5. \end{aligned}$$

At the one-loop level **the form factors** are given by

$$F_1^V = F_1^A = 1 - \frac{\alpha_s(q^2)}{4\pi} C_F \left(4 + \frac{1}{y}\ln(1-y) + \ln\left(\frac{y}{1-y}\frac{\Lambda^4}{(1-y)^2}\right) + 2\ln\left(\frac{\Lambda^2}{y}\frac{1}{1-y}\right)\ln\left(\frac{y}{1-y}\right) + 2Li(y)\right),$$

$$F_2^V = -F_2^A = \frac{1}{m_t}\frac{\alpha_s(q^2)}{4\pi} C_F \frac{2}{y}\left(+1 + \frac{1-y}{y}\ln(1-y)\right),$$

$$F_3^V = -F_3^A = \frac{1}{m_t}\frac{\alpha_s(q^2)}{4\pi} C_F \frac{2}{y}\left(-1 + \frac{2y-1}{y}\ln(1-y)\right),$$

where a gluon mass regulator $\Lambda = m_g/m_t$ is used to regularize the gluon IR singularity.

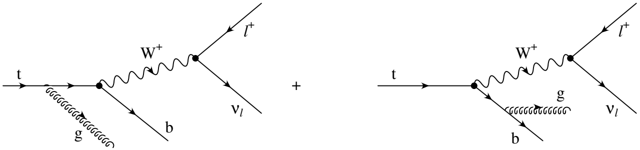
The hadron tensor for the **virtual correction** is

$$\begin{aligned} H_{\mu\nu} &= \sum_{s_b} H_\mu H_\nu^\dagger = \sum_{s_b} \left(H_\mu^{(0)} + H_\mu^{(1)}\right)\left(H_\nu^{(0)} + H_\nu^{(1)}\right)^\dagger \\ &= \sum_{s_b} H_\mu^{(0)} H_\nu^{(0)\dagger} + \sum_{s_b} \left(H_\mu^{(0)} H_\nu^{(1)\dagger} + H_\mu^{(1)} H_\nu^{(0)\dagger}\right) + \mathcal{O}(\alpha_s^2) \\ &= H_{\mu\nu}^{(0)} + H_{\mu\nu}^{(1)} + \mathcal{O}(\alpha_s^2) \end{aligned}$$

The $O(\alpha_s)$ virtual correction to the decay rate

$$\overline{|M^{(1)}|^2} = \sum_{s_b, s_\ell, s_\nu} |M^{(1)}|^2 \rightarrow \frac{G_F^2 |V_{tb}|^2}{2} H_{\mu\nu}^{(1)} L^{\mu\nu}$$

The real corrections



$$H_\mu = \bar{u}_b \left(\frac{\gamma_\mu \not{k} \gamma_\alpha - 2p_{t\alpha}\gamma_\mu}{2p_t \cdot k} + \frac{\gamma_\alpha \not{k} \gamma_\mu + 2p_{b\alpha}\gamma_\mu}{2p_b \cdot k} \right) (1 - \gamma_5)\, \epsilon_\lambda^{\alpha *} u_t.$$

The hadron tensor for the **real correction** is

$$
\begin{aligned}
\mathcal{H}^{\mu\nu} \quad = \quad & -4\pi\alpha_s\, C_F\, \frac{8}{(k\cdot p_t)(k\cdot p_b)} \Bigg\{ -\frac{k\cdot p_t}{k\cdot p_b} \Big[\\
& i\Big(\epsilon^{\alpha\beta\mu\nu}\,(p_b-k)\cdot\bar{p}_t - \epsilon^{\alpha\beta\gamma\nu}(p_b-k)^{\mu}\,\bar{p}_{t,\gamma} + \epsilon^{\alpha\beta\gamma\mu}(p_b-k)^{\nu}\,\bar{p}_{t,\gamma}\Big) k_\alpha\, p_{b,\beta}\Big] + \\
& \frac{k\cdot p_b}{k\cdot p_t}\Big[(\bar{p}_t\cdot p_t)\Big(k^\mu\, p_b^\nu + k^\nu\, p_b^\mu - k\cdot p_b\, g^{\mu\nu} - i\,\epsilon^{\alpha\beta\mu\nu} k_\alpha\, p_{b,\beta}\Big) \\
& -(\bar{p}_t\cdot k)\Big((p_t-k)^\mu\, p_b^\nu + (p_t-k)^\nu\, p_b^\mu - (p_t-k)\cdot p_b\, g^{\mu\nu} - i\,\epsilon^{\alpha\beta\mu\nu}(p_t-k)_\alpha\, p_{b,\beta}\Big)\Big] \\
& -(\bar{p}_t\cdot p_b)\Big(k^\mu\, p_b^\nu + k^\nu\, p_b^\mu - k\cdot p_b\, g^{\mu\nu} - i\,\epsilon^{\alpha\beta\mu\nu} k_\alpha\, p_{b,\beta}\Big) + (p_t\cdot p_b)\Big(k^\mu\,\bar{p}_t^\nu + k^\nu\,\bar{p}_t^\mu - k\cdot\bar{p}_t\, g^{\mu\nu}\Big) \\
& -(k\cdot p_b)\Big(p_t^\mu\,\bar{p}_t^\nu + p_t^\nu\,\bar{p}_t^\mu - p_t\cdot\bar{p}_t\, g^{\mu\nu}\Big) + (k\cdot p_t)\Big((p_b+k)^\mu\,\bar{p}_t^\nu + (p_b+k)^\nu\,\bar{p}_t^\mu - (p_b+k)\cdot\bar{p}_t\, g^{\mu\nu}\Big) \\
& +(k\cdot\bar{p}_t)\Big(2p_b^\mu\, p_b^\nu - p_b\cdot p_b g^{\mu\nu}\Big) + i\Big(\epsilon^{\alpha\beta\mu\nu}\,(k\cdot\bar{p}_t) + \epsilon^{\alpha\beta\gamma\mu}\, k^\nu\bar{p}_{t,\gamma} - \epsilon^{\alpha\beta\gamma\nu}\, k^\mu\bar{p}_{t,\gamma}\Big) p_{b,\alpha}\, p_{t,\beta} \\
& + i\Big(\epsilon^{\alpha\beta\mu\nu}\,(p_t\cdot\bar{p}_t) + \epsilon^{\alpha\beta\gamma\mu}\, p_t^\nu\bar{p}_{t,\gamma} - \epsilon^{\alpha\beta\gamma\nu}\, p_t^\mu\bar{p}_{t,\gamma}\Big) k_\alpha\, p_{b,\beta}\Bigg\} + B^{\mu\nu}\cdot\Delta_{SGF}
\end{aligned}
$$

$$
\begin{aligned}
\bar{p}_t^\mu &= p_t^\mu - m_t s_t^\mu \\
k &\rightarrow \text{gluon momentum} \\
\Delta_{SGF} &\rightarrow \textit{IR singular part} \\
B^{\mu\nu} &\rightarrow \text{the hadron tensor for the Born term.}
\end{aligned}
$$

Dalitz plot for the four-body phase space

$$
\begin{aligned}
dR_4\left(p_t; p_b, \ell, \nu, k\right) &= dP^2 dR_3\left(p_t; P, \ell, \nu\right) dR_2\left(P; p_b, k\right) \\
&= m_t^2 dz\, dR_2\left(P; p_b, k\right) dR_3\left(p_t; P, \ell, \nu\right)
\end{aligned}
$$

where $z = \frac{P^2}{m_t^2} = \frac{(p_b+k)^2}{m_t^2}$.

$O(\alpha_s)$ differential rates in lepton energy

$$\frac{d\Gamma_i^{(1)}}{dx_\ell dz} = \Gamma_F 2\pi \frac{M_W}{\Gamma_W} C_F \frac{\alpha_s}{2\pi} 6y \left[\underbrace{M_4^i(x_\ell, z)}_{real} + \underbrace{M_3^i(x_\ell)}_{virtual} \delta(z) \right], \; (i = A, B, C)$$

(the lepton momentum defines the z–axis)

$$\begin{aligned} M_4^C(x_\ell, z) &= \sqrt{y(1-x_\ell)(x_\ell - y) - x_\ell yz} \Bigg(\frac{y}{x_\ell \lambda^3} j_1 + \frac{1}{\lambda^3} j_2 + \frac{x_\ell}{\lambda^3} j_3 \\ &\quad + 4Y_P \left[\frac{6yz}{x_\ell \lambda^{7/2}} j_4 + \frac{1}{\lambda^{7/2}} j_5 + \frac{x_\ell}{\lambda^{7/2}} j_6 \right] \Bigg) \\ M_3^C(x_\ell) &= -\sqrt{y(1-x_\ell)(x_\ell - y)} \left(\frac{1-x_\ell}{y} \right) \ln(1-y), \end{aligned}$$

with $\lambda = 1 + y^2 + z^2 - 2(y + z + yz)$, $Y_P = \frac{1}{2} \log \frac{1-y+z+\sqrt{\lambda}}{1-y+z-\sqrt{\lambda}}$ and $j \sim f(y)$.

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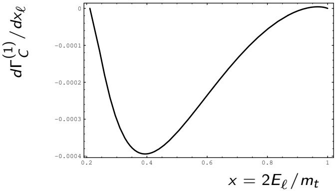
(the neutrino momentum defines the z–axis)

- Difficulties in obtaining **closed form** for $\frac{d\Gamma_C^{(1)}}{dx}$

$$\int dz \frac{z^n \sqrt{y(1-x)(x-y)-xyz}}{\sqrt[m]{1+y^2+z^2-2(y+z+yz)}} \log \frac{1-y+z+\sqrt{1+y^2+z^2-2(y+z+yz)}}{1-y+z-\sqrt{1+y^2+z^2-2(y+z+yz)}}.$$

- The numerical integration gives:



The closed form integrated rate $\Gamma_C^{(1)}$:

$$\Gamma_C = \int dx_{\ell,\nu} \int dz \frac{d\Gamma_C}{dx_{\ell,\nu} dz} = \int dz \int dx_{\ell,\nu} \frac{d\Gamma_C}{dx_{\ell,\nu} dz}.$$

NLO integrated azimuthal correlation function

$$\begin{aligned}\Gamma_C^{(1)} &= \Gamma_F\, 2\pi \frac{M_W}{\Gamma_W} C_F \left(-\frac{\alpha_s}{2\pi}\right) \frac{\pi}{8} y \Big[2\sqrt{y}(-4 - 3y + 3y^2)(2Li_2(\sqrt{y}) - Li_2(y)) \\ &\quad + 2(1-y)(8 - 7\sqrt{y} + 8y - 5y^{3/2}) \ln(1+\sqrt{y}) - \frac{(1-y)^3}{\sqrt{y}} \ln(1-y) \\ &\quad + \frac{1}{3}\sqrt{y}(6(1-\sqrt{y})^2(1-\sqrt{y}-2y) + \pi^2(4+3y-3y^2)) \Big].\end{aligned}$$

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Numerical results

Total rate with lepton momentum defined as the z–axis:

$$\frac{d\Gamma}{d\cos\theta\, d\phi} = \frac{1}{4\pi}\left[\Gamma_A + P(\Gamma_B \cos\theta + \Gamma_C \sin\theta\cos\phi)\right]$$

$$= \frac{\Gamma_A^{(0)}}{4\pi}\left[\left(1+\frac{\Gamma_A^{(1)}}{\Gamma_A^{(0)}}\right) + \left(1+\frac{\Gamma_B^{(1)}}{\Gamma_A^{(0)}}\right) P\cos\theta + \frac{\Gamma_C^{(1)}}{\Gamma_A^{(0)}} P\sin\theta\cos\phi\right]$$

$$= \frac{\Gamma_A^{(0)}}{4\pi}\left[(1-\underline{8.54}\,\%) + (1-\underline{8.71}\,\%)\, P\cos\theta - \underline{0.24}\%\, P\sin\theta\cos\phi\right]$$

- Roughly 9% corrections to the *unpolarized* and *polarized* rate.
- only 0.24% correction to the *azimuthal* rate with lepton on the z–axis .

Total rate with neutrino momentum defined as the z–axis:

$$\frac{d\Gamma}{d\cos\theta\, d\phi} = \frac{\Gamma_A^{(0)}}{4\pi}\left[(1-\underline{8.54}\,\%) + (-0.318+\underline{1.02}\,\%)\, P\cos\theta + (0.919-\underline{0.029}\,\%)\, P\sin\theta\cos\phi\right]$$

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Conclusions

For the decay $t(\uparrow) \to b + W^+(\ell^+ \nu)$:

- Very small NLO contribution to the azimuthal correlation (in the lepton case).
- Precision measurements for the azimuthal correlation can be important in understanding the SM.