Advanced Theoretical Astro–Particle Physics (WS 22/23)
Homework no. 1 (October 12, 2022)

To be completed by: Thursday, October 20.

1 Muons in Cosmic Rays

Muons are the most common charged CR component at ground level; as stated in class, their flux at $E_\mu \geq 1$ GeV amounts to about 70 $\mathrm{m}^{-2}\ \mathrm{s}^{-1}\ \mathrm{sr}^{-1}$.

1. Given that most muons are created at an altitude of about 15 km, what is the minimal energy a muon needs to have in order to reach the ground? How does this depend on the incidence angle θ (measured relative to the horizon)? *Hint:* The lifetime of the muon is about 2 microseconds.

2. In class we saw that the spectrum of CR primaries at "low" energy is approximately given by

$$I_N \equiv \frac{d\Phi_N}{dE_N} \simeq 1.8 \cdot 10^4 \left(\frac{E_N}{1\ \mathrm{GeV}}\right)^{-2.7}\ \mathrm{m}^{-2}\ \mathrm{s}^{-1}\ \mathrm{sr}^{-1}\ \mathrm{GeV}^{-1}\,. \tag{1}$$

   This is the flux of nucleons, most of which occur in the form of free protons in this energy range. Estimate the typical energy of a proton that produces (about) 1 muon at sea level.

2 Production of Higgs Bosons

The SM Higgs boson h of mass 125 GeV can be produced in pp (or nucleon–nucleon) collisions primarily via gluon–gluon fusion. The total production cross section can *roughly* be parameterized as

$$\sigma(pp \to h) \simeq 50\,\mathrm{pb} \left(\frac{\sqrt{s}}{13\ \mathrm{TeV}}\right)^{1.7}\ ; \tag{2}$$

the value of the cross section at the LHC energy of 13 TeV is quite accurate, the energy dependence less so. Here $\sqrt{s}$ is the pp center–of–mass energy, s being one of the Mandelstam variables.

1. Rewrite eq.(2) in terms of the equivalent energy of a proton (or nucleon) hitting another nucleon at rest, as e.g. CR nucleons colliding with nucleons in air.

2. In the relevant energy range, the total spectrum of CR primaries is roughly given by

$$I = 1.6 \cdot 10^{-18} \left(\frac{E}{10^8\ \mathrm{GeV}}\right)^{-3.3}\ \mathrm{m}^{-2}\ \mathrm{s}^{-1}\ \mathrm{sr}^{-1}\ \mathrm{GeV}^{-1}\,. \tag{3}$$

Assuming these are all free protons, estimate the *total* rate of Higgs bosons produced by CRs impinging on the Earth's atmosphere. How does this compare to the production rate at the LHC, with luminosity of about 100 fb^{-1} per year? *Hints:* Use eq.(2) for the cross section at $\sqrt{s} \geq 3$ TeV, and set the cross section at even smaller energies to zero. Assume that only the first interaction can produce a Higgs boson (this should be a very good approximation). The total pp cross section is about 100 mb. The radius of the Earth is about 6,400 km.

3. How would the result change if eq.(3) instead describes the flux of some heavier nucleus, with mass number A? *Hint:* Recall that E in eq.(3) is the energy of the entire nucleus, but only a single nucleon takes part in a given "hard" collision that can produce a Higgs boson.