Advanced Theoretical Astro–Particle Physics (WS 22/23)
Homework no. 10 December 22, 2022)

To be completed by: Thursday, January 12, 2023.

1 $J-$Factor

The flux from the annihilation of Dark Matter (DM) particles in some region of space is proportional to the $J-$factor of that region, defined by

$$J = \int_0^\infty dl \int_{\Delta\Omega} d\Omega \, \rho_\chi^2(l, \Omega) \,. \tag{1}$$

Here ρ_χ is the DM mass density, l is along the line of sight, and the angular interval $\Delta\Omega$ should at least cover the angular resolution of the instrument.

1. We want to evaluate the contribution to eq.(1) for a spherically symmetric DM distribution centered on the line of sight. Assume that the radius $\bar{R}$ of the distribution (defined either by the opening angle chosen, or by the true size of the distribution) is much smaller than the distance d to the center of the distribution, and show that J can then be written as
$$J = \frac{4\pi}{d^2} \int_0^{\bar{R}} r^2 \rho_\chi^2(r) \, dr \,. \tag{2}$$

2. Show that eq.(2) can be rewritten as
$$J = \frac{M_\chi(\bar{R}) \bar{\rho}_\chi}{d^2} \,, \tag{3}$$
where $M_\chi(\bar{R})$ is the total DM mass inside radius $\bar{R}$ and $\bar{\rho}_\chi$ is the *average* DM density in the considered region.

3. The DM density is frequently parameterized as a generalized Navarro–Frenk–White (gNFW) distribution:
$$\rho_\chi(r) = \frac{\rho_0}{(r/R)^\gamma \left[1 + (r/R)^\alpha\right]^{(\beta-\gamma)/\alpha}} \,. \tag{4}$$
This is evidently spherically symmetric, r being the distance from the center of the distribution. Evaluate the $J-$factor for the center of our own galaxy, which is (about) 8.5 kpc away, for a half–angle of 3° (i.e. the opening angle of the observed region is 6°), for the following two cases: *(i)* original NFW: $\alpha = \gamma = 1, \ \beta = 3, \ R = 20$ kpc; *(ii)* isothermal: $\alpha = \beta = 2, \ \gamma = 0, \ R = 3.5$ kpc. *Hint:* Note that $r \ll R$ holds for the entire integral, i.e. the $r \ll R$ limit of eq.(4) can be used in eq.(2). Also, use the fact that the local DM density is (approximately) 0.4 GeV/cm^3 to determine ρ_0 for these two cases.

2 Dark Matter Decaying into Neutrinos

In this exercise we consider $\chi \to \nu\bar{\nu}$ decays. (Evidently the DM particle χ must be a boson.)

1. Show that in this case the flux of neutrinos from a spherically symmetric distribution with radius $\bar{R}$ much smaller than the distance d is proportional to the modified $\tilde{J}$ factor
$$\tilde{J} = \frac{4\pi}{d^2} \int_0^{\bar{R}} r^2 \rho_\chi(r) dr \,. \tag{5}$$

2. Evaluate $\tilde{J}$ for the innermost 1 kpc of our galaxy, for the NFW distribution as defined above.

3. Show that the flux from χ particles decaying nearby, i.e. in a sphere of radius R_0 centered on the solar system, is proportional to
$$\tilde{J}_{\rm sph} = 4\pi \int_0^{R_0} \rho_\chi(r_s) dr_s \,, \tag{6}$$
where r_s is the distance to the solar system (*not* from the center of the DM distribution). Evaluate this contribution for $R_0 = 1$ kpc, assuming a constant ρ_χ with the value given in the previous problem. Show that this contribution exceeds that from the inner galaxy.

4. Show that the differential $\nu + \bar{\nu}$ flux per angle from nearby χ decays is given by
$$\frac{d\phi_{\nu+\bar{\nu}}}{dE} = \frac{\rho_\chi}{2\pi m_\chi \tau_\chi} \delta(E - m_\chi/2) \,, \tag{7}$$
where τ_χ is the lifetime of χ. Estimate the lower bound on this lifetime for two cases: *(i)* $m_\chi = 1$ PeV, integrated flux $\phi < 0.6 \cdot 10^{-12}/(\text{cm}^2 \cdot \text{s} \cdot \text{sr})$; *(ii)* $m_\chi = 1$ TeV, integrated flux $\phi < 10^{-7}/(\text{cm}^2 \cdot \text{s} \cdot \text{sr})$. *Hint:* The upper bounds are rough estimates from IceCube observations. The true bounds can be made stronger by not just limiting the total flux, but limiting the normalization of a template flux with nontrivial angular dependence (being mildly peaked towards the galactic center).