Advanced Theoretical Astro–Particle Physics (WS 22/23)
Homework no. 11 (January 11, 2023)

To be completed by: Thursday, January 19, 2023.

1 Annihilation Just Below a Resonance

The averaged annihilation cross section of Dark Matter particles can be written as

$$\langle \sigma v \rangle = N(v_0) \int_0^\infty dv\, v^2 (\sigma v)\, \mathrm{e}^{-v^2/v_0^2}\,, \tag{1}$$

where a Gaussian distribution in the relative cms velocity v has been assumed, with typical velocity v_0.

1. Show that

$$N(v_0) = \frac{4}{v_0^3 \sqrt{\pi}}\,, \tag{2}$$

   from the requirement that the average of a constant (velocity–independent) $\sigma v = \Sigma$ =const should be given by Σ.

2. Here we are interested in WIMP annihilation very close (and just below) an $s-$channel resonance. Let m_M and Γ_M be the mass and width of the resonance (i.e. of the particle mediating the annihilation). Show that for *(i)* annihilation of scalar WIMPs through a scalar resonance into an SM $f\bar{f}$ pair; *(ii)* annihilation of Dirac WIMPs through a vector resonance into an SM $f\bar{f}$ pair the cross section for $v \ll 1$ (i.e. for non–relativistic WIMPs) can be written as

$$\sigma v \simeq \frac{\sigma_0}{\left(v^2 - 8\delta\right)^2 + \gamma^2}\,. \tag{3}$$

   Here $\sigma_0 \to$const. as $v \to 0$, and the dimensionless quantities δ and γ are defined via

$$m_M = 2m_\chi(1+\delta)\,, \quad \gamma = \frac{\Gamma_M}{m_M}\,, \tag{4}$$

   m_χ being the mass of the WIMP.

3. We are interested in scenarios where both δ and γ are much less than 1. Show that in this case

$$\langle \sigma v \rangle \propto \frac{\sqrt{\delta}}{\gamma v_0^3} \tag{5}$$

   for $v_0^2 \gtrsim 8\delta$, while

$$\langle \sigma v \rangle \propto \frac{1}{\delta^2} \tag{6}$$

   for $v_0^2 \ll 8\delta$; in the latter case the result is independent of γ (as long as $\gamma \lesssim \delta$) and v_0.