Advanced Theoretical Astro–Particle Physics (WS 22/23)

Homework no. 12 (January 18, 2023)

To be completed by: Thursday, January 26, 2023.

1 Supernova Type 1

A carbon/oxygen (CO) white dwarf (WD) with mass $M_{\rm WD} = 1 M_\odot = 2 \cdot 10^{33}$ g has a radius of about 7,000 km. Show that a CO WD with 1.4 solar masses can be destroyed completely by (explosive) fusion of carbon and oxygen into (eventually) iron and nickel, giving rise to a type–1 supernova. *Hint:* These final stages of nuclear "burning" release around 0.8 MeV per nucleon. Recall also that the radius of a WD scales like $R_{\rm WD} \propto M_{\rm WD}^{-1/3}$, see eq.(II.12).

2 Elastic Neutrinos Scattering on Iron

The core of a massive star that is about to explode as a type 2 supernova consists mostly of iron. As described in class, once it starts to collapse the core heats up; it keeps collapsing since some of this thermal energy is emitted by neutrinos. (Some more thermal energy is "used up" by dissociating iron nuclei.) Eventually the core gets so dense that neutrinos produced in its center cannot escape freely any more, but get scattered many times, i.e. diffuse, until they reach the outer part of the core, the "neutrino sphere", from where they can be emitted freely. Here we want to estimate the core radius, and hence density, where a neutrino sphere first forms.

The dominant reaction that can prevent neutrinos from escaping freely is elastic scattering on iron nuclei; let's consider ^{56}Fe for definiteness, which is a spinless nucleus consisting of 26 protons and 30 neutrons.

1. In general, neutral current (NC) scattering of neutrinos on nucleons involves vector and axial vector contributions. Show that in the limit where the momentum exchange q is much less than the nucleon mass m_N the axial vector contribution to the scattering on a spinless even–even nucleus (where the numbers of protons and neutrons are both even) is strongly suppressed, and can hence be ignored. *Hint:* In such a nucleus all neutrons and all protons are paired up in spin–singlet states.

2. The NC strength can be written as product of a coupling $g_Z = e/(2 \sin\theta_W \cos\theta_W)$, θ_W being the weak mixing angle, and a "weak charge" $Q_W = T_3 - 2\sin^2\theta_W Q_{\rm em}$, where T_3 is the third component of weak isospin and $Q_{\rm em}$ is the usual electromagnetic charge. Compute the weak charge of ^{56}Fe, and use this to write down the the matrix element for elastic $\nu + {\rm Fe} \to \nu + {\rm Fe}$ scattering. *Hint:* Neglect the neutrino masses, and take a neutrino energy $E_\nu \ll M_Z$, $m_{\rm Fe}$, where M_Z is the mass of the Z boson exchanged in this reaction. Also, convince yourself that for $E_\nu \lesssim 10$ MeV the Fe nucleus can

indeed be considered pointlike, by estimating its radius from the (very simple) ansatz $r_A = 1\ \text{fm} \cdot A^{1/3}$, A being the mass number of the nucleus (i.e. $A = 56$ for the case at hand).

3. Compute the total cross section from the previous result, again in the limit $E_\nu \ll m_{\text{Fe}}$, assuming purely elastic scattering (outgoing neutrino energy $E'_\nu = E_\nu$). A necessary condition for the formation of a neutrino sphere is that the corresponding neutrino interaction length $\lambda_\nu = 1/(\sigma n_{\text{Fe}})$ should be less than the radius R_C of the core. Solve this condition for R_C, assuming that the collapsing core weighs about one solar mass $= 2 \cdot 10^{33}$ g.

4. Give two reasons why this simple calculation gives a significantly smaller critical core density than the value of about $10^{12}\,\text{g/cm}^3$ (for $E_\nu = 10$ MeV) cited for the onset of the formation of the neutrino sphere in class. *Hint:* One reason is related to the angular dependence of the scattering cross section.