Theoretical Astro–Particle Physics (SS 25) Homework no. 3 (April 24, 2025)

## 1 "Reheating" by Particle Decay

Consider a massive, unstable particle X with mass  $m_X$  and lifetime  $\tau_X$ , such that the energy density of the Universe can be written as a sum  $\rho_{\text{tot}} = \rho_R + \rho_X$ , where  $\rho_R$  is the radiation energy density due to (effectively stable) ultrarelativistic particles of the Standard Model. In this exercise we want to see how the decay of X changes the radiation. To this end, we employ the "instantaneous decay approximation", where all X-particles decay at temperature  $T = T_d \ll m_X$ , i.e. the X-particles are non-relativistic when they decay. Moreover, we assume that the decay products of X thermalize instantaneously, i.e. contribute to  $\rho_R$ .

- 1. Argue that this decay should leave  $\rho_{tot}$  unchanged. This means that it *increases* the entropy. Why is this possible?
- 2. Calculate the increase of the temperature of the thermal bath, i.e. calculate the ratio  $T_a/T_d$  where  $T_a$  is the temperature just after X-decay.
- 3. Use the result of the previous step to compute the increase of the entropy density,  $s_a/s_d$ .
- 4. Now assume that X "decoupled" at temperature  $T_X \gg T_d$ , with  $n_X(T_X) = c_X T_X^3$ (where the constant  $c_X$  is roughly of order unity) and  $n_X(T)/s(T) = \text{const.}$  for  $T_d < T < T_X$ . Show that

$$s_a/s_d \propto m_X c_X \sqrt{\tau_X},$$

independent of  $T_X$ , if  $\rho_X \gg \rho_R$  for most of the time between the decoupling and decay of X.

5. How does  $s_a/s_d$  scale with  $m_X$  for fixed couplings of X, if X undergoes (i) 2-body decay, (ii) 3-body decay via the exchange of a boson Y with  $m_Y \gg m_X$ ? *Hint:* use the relation between  $m_X$  and  $\tau_X$ !

Note: The entropy increase factor estimated in this simple manner accurately reproduces the exact result at times  $t \gg \tau_X$ . However, in reality a particle decaying in the expanding universe does not increase the temperature of the thermal bath (unless  $\rho_X \gg \rho_R$ already at temperature  $T_X$ , which is impossible for particles that once were in thermal equilibrium); it merely slows down the rate of decrease. Hence "reheating" in the title is put in quotation marks.

## 2 Superparticles in the Early Universe

The notion of supersymmetry postulates that every SM particle has a superpartner with spin differing by half a unit, but equal gauge quantum numbers; that is, for each SM fermion of given chirality there should exist a complex "sfermion", and for each SM gauge boson a spin-1/2 Majorana "gaugino". In addition, a second complex Higgs doublet, plus fermionic (spin-1/2) superpartners, needs to be introduced. Altogether there should thus be an equal number of fermionic and bosonic degrees of freedom in a supersymmetric world; more generally, a supersymmetric theory should be invariant when an SM particle is replaced by its superpartner.

- 1. Show by explicit calculation that at temperatures well above the mass of all particles (SM particles and superpartners), this increases the number of degrees of freedom from 106.75 to 228.75. Why does this number increase by more than a factor of 2?
- 2. How does this change the relation between time and temperature, for  $T \gg m_{\text{SUSY}}$ , where  $m_{\text{SUSY}}$  is the superpartner mass scale?
- 3. Argue that the thermal bath breaks supersymmetry.