Advanced Theoretical Astro–Particle Physics (WS 22/23)

Homework no. 3 (October 26, 2022)

To be completed by: Thursday, November 3.

1 Production of ^{14}C in Air

The Carbon–14 timing method, frequently used in archaeology, only works because the concentration of ^{14}C in the atmosphere remains approximately constant. In this exercise we look at the main processes that determine the concentration of ^{14}C atoms (or nuclei) in the atmosphere.

1. ^{14}C undergoes $\beta-$decay,
$$^{14}\text{C} \to {}^{14}\text{N} + e^- + \bar{\nu}_e \,. \tag{1}$$
The lifetime of about 5,700 years determines the range of archaeological ages that can be determined by measuring the $^{14}\text{C}/^{12}\text{C}$ ratio in the sample; the lifetime is large mostly because the $Q-$value is small,
$$m(^{14}\text{C}) - m(^{14}\text{N}) = 0.28 \text{ MeV} \,. \tag{2}$$
On the other hand, the main process for producing ^{14}C in air is
$$n + {}^{14}\text{N} \to p + {}^{14}\text{C} \,. \tag{3}$$
What is the minimal energy the neutron n must have for this reaction to proceed?

2. Argue that the density n_C of ^{14}C nuclei in the atmosphere has the following time dependence:
$$\frac{dn_\text{C}(X)}{dt} = -\Gamma_\text{C} n_\text{C}(X) + 2\pi\, n_\text{N}(X) \int_0^1 d\cos\theta \int_{E_\text{min}}^\infty dE_n \sigma(E_n) F_n(E_n, X/\cos\theta) \,. \tag{4}$$
Here n_N is the density of ^{14}N nuclei, σ is the cross section for reaction (3), F_n is the differential flux of neutrons in the units used in class, and X is the (vertical) depth in the shower. Where does the factor 2π come from? Why does the integral over $\cos\theta$ start at 0 rather than 1, and where does the $\cos\theta$ dependence in the second argument of F_n come from?

3. In equilibrium the integral $\int_0^{X_\text{max}} dX n_\text{C}(X)$ must be constant, where $X_\text{max} \simeq 10^3$ g/cm^2. Why is the integration over X necessary? Use the equilibrium condition to rewrite this column integral, using eq.(4).

4. The entire atmosphere of Earth contains only about 850 kg of ^{14}C (according to Wikipedia). Compute the ratio

$$R_{\mathrm{C/N}} = \frac{\int_0^{X_{\max}} n_{\mathrm{C}}(X)}{\int_0^{X_{\max}} n_{\mathrm{N}}(X)} \,. \tag{5}$$

   Hint: Nitrogen contributes about 80% to the weight of the atmosphere.

5. The cross section for reaction (3) amounts to about 10^{-26} cm^2 = 10 mb. (It has been measured only at rather low energies, but let's assume it to be constant.) Use this, and the numerical value of $R_{\mathrm{C/N}}$, to estimate the average flux $\int dE_n F_n(E_n)$. You'll find it to be considerably smaller than the flux of protons at the top of the atmosphere. Give some reason(s) for this. *Hint:* The real cross section should decline quickly with neutron energy, at least for relativistic neutrons. What can happen to non–relativistic neutrons in air?

6. Starting in about 1750 A.D., human activity reduced the ^{14}C to ^{12}C ratio in the atmosphere; between about 1950 and 1980, another human activity increased the amount of ^{14}C in the atmosphere. What are these activities?