

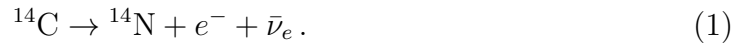
Advanced Theoretical Astro-Particle Physics (WS 22/23)
Homework no. 3 (October 26, 2022)

To be completed by: Thursday, November 3.

1 Production of ^{14}C in Air

The Carbon-14 timing method, frequently used in archaeology, only works because the concentration of ^{14}C in the atmosphere remains approximately constant. In this exercise we look at the main processes that determine the concentration of ^{14}C atoms (or nuclei) in the atmosphere.

1. ^{14}C undergoes β -decay,



The lifetime of about 5,700 years determines the range of archaeological ages that can be determined by measuring the $^{14}\text{C}/^{12}\text{C}$ ratio in the sample; the lifetime is large mostly because the Q -value is small,

$$m(^{14}\text{C}) - m(^{14}\text{N}) = 0.28 \text{ MeV}. \quad (2)$$

On the other hand, the main process for producing ^{14}C in air is



What is the minimal energy the neutron n must have for this reaction to proceed?

2. Argue that the density n_C of ^{14}C nuclei in the atmosphere has the following time dependence:

$$\frac{dn_C(X)}{dt} = -\Gamma_C n_C(X) + 2\pi n_N(X) \int_0^1 d\cos\theta \int_{E_{\min}}^{\infty} dE_n \sigma(E_n) F_n(E_n, X/\cos\theta). \quad (4)$$

Here n_N is the density of ^{14}N nuclei, σ is the cross section for reaction (3), F_n is the differential flux of neutrons in the units used in class, and X is the (vertical) depth in the shower. Where does the factor 2π come from? Why does the integral over $\cos\theta$ start at 0 rather than 1, and where does the $\cos\theta$ dependence in the second argument of F_n come from?

3. In equilibrium the integral $\int_0^{X_{\max}} dX n_C(X)$ must be constant, where $X_{\max} \simeq 10^3$ g/cm². Why is the integration over X necessary? Use the equilibrium condition to rewrite this column integral, using eq.(4).

4. The entire atmosphere of Earth contains only about 850 kg of ^{14}C (according to Wikipedia). Compute the ratio

$$R_{\text{C/N}} = \frac{\int_0^{X_{\text{max}}} n_{\text{C}}(X) dX}{\int_0^{X_{\text{max}}} n_{\text{N}}(X) dX}. \quad (5)$$

Hint: Nitrogen contributes about 80% to the weight of the atmosphere.

5. The cross section for reaction (3) amounts to about $10^{-26} \text{ cm}^2 = 10 \text{ mb}$. (It has been measured only at rather low energies, but let's assume it to be constant.) Use this, and the numerical value of $R_{\text{C/N}}$, to estimate the average flux $\int dE_n F_n(E_n)$. You'll find it to be considerably smaller than the flux of protons at the top of the atmosphere. Give some reason(s) for this. *Hint:* The real cross section should decline quickly with neutron energy, at least for relativistic neutrons. What can happen to non-relativistic neutrons in air?
6. Starting in about 1750 A.D., human activity reduced the ^{14}C to ^{12}C ratio in the atmosphere; between about 1950 and 1980, another human activity increased the amount of ^{14}C in the atmosphere. What are these activities?