Theoretical Astro–Particle Physics (SS 25) Homework no. 7 (June 5, 2025)

1 WIMP–Nucleus Scattering Cross Section

In this exercise we want to compute the cross section relevant for "direct" detection of Dark Matter WIMPs χ , which is based on the elastic scattering of WIMPs off nuclei in a detector.

1. Let the effective Lagrangian be given by

$$\mathcal{L}_{\text{eff}} = \sum_{N=p,n} \left(f_N \bar{\chi} \chi \bar{N} N + g_N \bar{\chi} \gamma_\mu \gamma_5 \chi \bar{N} \gamma^\mu \gamma_5 N \right) \,. \tag{1}$$

Here N stands for a nucleon, χ for a spin-1/2 Majorana WIMP, and the coefficients $f_{n,p}$, $g_{n,p}$ have units GeV⁻². (Why?) Calculate the resulting matrix element for $\chi N \to \chi N$ scattering. Use explicit expressions for the Dirac spinors rather than trace techniques; this makes the physical meaning of the two terms more transparent. *Hints:* Since ambient WIMPs are assumed to be quite slow, $v_{\chi} < 10^3$ km/s, you can use non-relativistic expansions of the relevant spinors; ignore terms in the scattering matrix element that are of second (or higher) order in 3-momenta. The Dirac representation is then more convenient than the chiral representation of the spinors and γ -matrices, with $\phi_f^{\dagger} \vec{\sigma} \phi_f \to 4m_f \vec{S}_f$ for spin-1/2 fermion f, σ_i being the Pauli matrices, ϕ_f the large component of the 4-spinor u_f and \vec{S}_f the spin.

- 2. Use the results of the previous step to compute the total cross section for elastic $\chi N \to \chi N$ scattering. *Hints:* Prove and use the quantum mechanical relation $\vec{S_{\chi}} \cdot \vec{S_N} = 0.5[S(S+1) 3/2]$, where S is the total spin quantum number of the system; when computing the squared matrix element, remember the multiplicities of the two possible spin states, i.e. the two possible values of S. Also, recall the standard result (allowing for massive particles in the initial state!) for the differential cross section $d\sigma/d\Omega^*$, where $d\Omega^*$ is the angular phase space element in the center-of-mass frame.
- 3. Now consider the case where the WIMPs scatter off a nucleus with mass number A. Argue that this gives an extra factor of A enhancement to the term $\propto f_N$ in the $\chi A \to \chi A$ amplitude, if $f_n = f_p$ (which is true in many WIMP models, up to small corrections), whereas no large enhancement results for the terms $\propto g_N$. Under what conditions do the latter vanish? *Hint:* You can ignore form factor effects, i.e. the nucleus is heavy but point-like.
- 4. Finally, compute the cross section for $\chi A \to \chi A$ scattering, including only the term $\propto |f_N|^2$.

2 Flux of Dark Matter Particles on Earth

Estimate the flux of dark matter particles on Earth, assuming a local dark matter mass density of 0.4 GeV/cm³ and a typical velocity of a dark matter particle equal to the local galactic rotational velocity, which is about 220 km/s. For a typical WIMP of mass 100 GeV, how does this flux compare with the flux of cosmic ray muons, which is of order $1/(m^2 \cdot s)$? What if dark matter instead consists of primordial black holes of mass 10^{17} g? *Hint:* Only order of magnitude estimates are needed here.