

Collider Physics (WS 26/27)
Homework no. 1 (October 14, 2026)

1) Lorentz Invariance of the ϵ tensor

In class we introduced the totally antisymmetric rank-4 tensor, which is anti-symmetric under the exchange of any two of its indices, and satisfies

$$\epsilon^{0123} = -\epsilon_{0123} = 1. \quad (1)$$

Show that this tensor is invariant (i) under a spatial rotation, (ii) under a Lorentz boost. *Hint:* For an explicit calculation, fix the axis of rotation and the boost direction to coincide with one of the coordinate axes. Show that ϵ_{0123} (or ϵ^{0123}) transforms into itself, and that the transformed tensor vanishes if two indices are equal.

2) Mandelstam variables

Consider a $2 \rightarrow 2$ scattering reaction, $1 + 2 \rightarrow 3 + 4$. Energy-momentum conservation implies

$$p_1 + p_2 = p_3 + p_4 \quad (2)$$

where the p_i are 4-momenta. The Mandelstam variables are defined by [see Eqs.(1.22) shown in class]:

$$s = (p_1 + p_2)^2; \quad t = (p_1 - p_3)^2; \quad u = (p_1 - p_4)^2. \quad (3)$$

1. Show that Eq.(2) implies Eq.(1.23), i.e.

$$s + t + u = \sum_{i=1}^4 m_i^2, \quad (4)$$

where $m_i^2 = p_i^2 \equiv p_i \cdot p_i$ are the squared (rest) masses of the particles.

2. Let E_i be the energy of particle i in the center-of-mass frame (where $\vec{p}_1 + \vec{p}_2 = \vec{p}_3 + \vec{p}_4 = 0$). Show that the E_i are given by Eqs.(1.25), i.e.

$$\begin{aligned} E_1 &= \frac{s + m_1^2 - m_2^2}{2\sqrt{s}}; & E_2 &= \frac{s + m_2^2 - m_1^2}{2\sqrt{s}}; \\ E_3 &= \frac{s + m_3^2 - m_4^2}{2\sqrt{s}}; & E_4 &= \frac{s + m_4^2 - m_3^2}{2\sqrt{s}}. \end{aligned} \quad (5)$$

3. In class we computed the minimal and maximal values of t and u for the important special case $m_1 = m_2 = 0$, $m_3 = m_4 \neq 0$. We found that in this case both t and u are always negative. Show that for general combinations of masses t can be positive, if $(m_1 - m_3)(m_4 - m_2) > 0$. What happens if $s \gg m_i^2 \forall i$?

4. Show that $t \geq -s$ holds for all combinations of masses.
5. Consider deep-inelastic electron-proton scattering, $e + p \rightarrow e + X$, where X is a system of particles with unknown “mass” (i.e. p_X^2 is not fixed). Compute the maximal 4-momentum transfer $Q^2 \equiv -t = (p_e - p'_e)^2$ as function of p_X^2 , where p'_e is the 4-momentum of the outgoing electron, for given electron energy E_e in the proton rest frame. *Hint:* Ignore the electron mass.