

Collider Physics (WS 26/27)

Homework no. 2 (October 21, 2026)

Tutorials: Monday, 16:15 to 17:45, ROT, seminar room (downstairs)

1) Three-body kinematics

In class we introduced the description of the kinematics of a system of three particles with 4-momenta p_i in terms of the Dalitz variables, $m_{ij}^2 = (p_i + p_j)^2$, $i, j \in \{1, 2, 3\}$.

1. Prove that if both the total cms energy $\sqrt{s}$ and the cms energy E_1 of the first particle are fixed, the minimal and maximal values of m_{12}^2 are given by [cf. (1.48)]

$$m_{12,max,min}^2 = (E_1^* + E_2^*)^2 - (|\vec{p}_1^*| \mp |\vec{p}_2^*|)^2 ; \quad (1)$$

here the E_i^* refer to energies in the (23) rest frame, defined by $\vec{p}_2^* + \vec{p}_3^* = 0$. *Hint:* In class we had shown that $m_{12}^2 = m_1^2 + m_2^2 + 2(E_1^* + E_2^* - \vec{p}_1^* \cdot \vec{p}_2^*)$, and that E_1^* and E_2^* are independent of the angle between $\vec{p}_1^*$ and $\vec{p}_2^*$.

2. How are the three momentum vectors $\vec{p}_1$, $\vec{p}_2$ and $\vec{p}_3$ oriented, when (i) m_{12} takes its absolute minimum; (ii) m_{12} takes its absolute maximum; (iii) m_{12} takes its maximum for fixed m_{23} ; (iv) m_{12} takes its minimum for fixed m_{23} . In cases (i) and (ii) also give the ratio $|\vec{p}_1|/|\vec{p}_2|$. *Hint:* Consider the last two configurations first in the (23) rest frame, before going back to the overall (123) rest frame.
3. Consider the case $m_1 = 0$. Why do m_{12} and m_{13} reach their minimal value simultaneously? What happens with m_{23} at this point?
4. Discuss the limits of the physical region of the Dalitz plane for the case $m_1 = m_2 = 0, m_3 \equiv m \neq 0$, which describes e.g. $e^+e^- \rightarrow Z\gamma\gamma$ at LEP. Consider both the (m_{12}^2, m_{13}^2) plane and the (m_{13}^2, m_{23}^2) plane.

2) Flux factor

The “flux factor” appearing in the expression for the cross section, derived in class [see eqs.(1.68), (1.69)] is given by

$$\mathcal{F} = 4E_A E_B |v_{z,A} - v_{z,B}| , \quad (2)$$

where it has been assumed that the beam particles travel along the z axis. Here $E_{A,B}$ are the energies of the two colliding particles, and $v_{z(A,B)}$ their velocities; the latter are given by the ratio of the z -component of the 3-momentum and the energy,

$$v_{z(A,B)} = \frac{p_{z(A,B)}}{E_{A,B}} . \quad (3)$$

1. Show that $\mathcal{F}$, with the velocities given by (3), is invariant under boosts along the z axis.
2. Is $\mathcal{F}$ invariant under boosts in x or y direction? Interpret the result.
3. Evaluate $\mathcal{F}$ in the rest frame of particle B .
4. Now consider the cms frame. Show that $\mathcal{F}$ can be expressed in terms of Mandelstam- s and the masses $m_{A,B}$ of the colliding particles. Compare with the result in the rest frame of particle B .
5. In question 2.4 we expressed $\mathcal{F}$ entirely in terms of Lorentz invariant quantities. Is this compatible with the result of question 2.2?