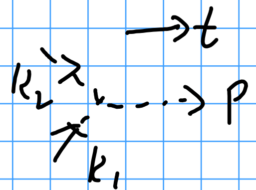
ϕ^3 theory ($\kappa = \lambda = 0$, $A \neq 0$ in (2.21), (2.23))

Here: $V_{KG} = A\phi$: not enough ϕ-factors to describe $2\to 2$ scattering directly from (2.26)!

Note: Can produce "$2\to 1$" scattering, but the produced particle cannot be "on-shell" ($p^2 \neq m^2$): is virtual particle

Want $k_1 + k_2 = p$



(I)

$k_1^2 = k_2^2 = m_\phi^2$

E.g. cms frame: $k_1^* = (E^*, 0, 0, |\vec{k}^*|)$, $k_2^* = (E^*, 0, 0, -|\vec{k}^*|)$,

$E^{*2} = |\vec{k}^*|^2 + m_\phi^2 \quad \Rightarrow p^2 = (k_1+k_2)^2 = (2E^*, \vec{0})^2 = 4E^{*2}$

$\Rightarrow p^2 = 4E^{*2} = 4(m_\phi^2 + |\vec{k}^*|^2) > m_\phi^2$!

But: can describe $2\to 2$ via "production" and "decay" (better: exchange) of virtual particle.

To produce ϕ_p to 1st order in pert. theory:

$$A_{fi}^{(I)} = -iA \int d^4x\, \phi_{k_1}(x)\, \phi_{k_2}(x)\, \phi_p^*(x) \qquad (2.45)$$

drop V factors

ϕ_p must satisfy e.o.m. with final state as "source":

$$(2.23) \Rightarrow \partial^\mu \partial_\mu \phi_p + m_\phi^2 \phi_p = -A\, \phi_{p_1} \phi_{p_2} = -A\, e^{-ix\cdot(p_1+p_2)} \qquad (2.46)$$

combinatorics factor $2! = 2$

Ansatz: $\phi_p = f(p)\, e^{-ip\cdot x}$

$$\Rightarrow (\partial_\mu \partial^\mu + m_\phi^2)\phi_p = (-p^2 + m_\phi^2)\, f(p)\, e^{-ip\cdot x} \overset{(2.46)}{=} -A\, e^{-ix\cdot(p_1+p_2)}$$

$\Rightarrow p = p_1 + p_2 = k_1 + k_2$ (4-mom. cons.)

$$f(p) = \frac{A}{p^2 - m_\phi^2} \qquad (2.47)$$

Insert into (2.45) ("iterated 1st order pert. th.")

$$A_{fi}^{(I)} = -iA^2 \int d^4x\, \phi_{k_1}(x)\, \phi_{k_2}(x)\, \frac{1}{p^2 - m_\phi^2}\, \phi^*_{p_1}(x)\, \phi^*_{p_2}(x)$$

$$= -iA^2 \frac{1}{p^2 - m_\phi^2} (2\pi)^4 \delta^{(4)}(k_1 + k_2 - p_1 - p_2)$$

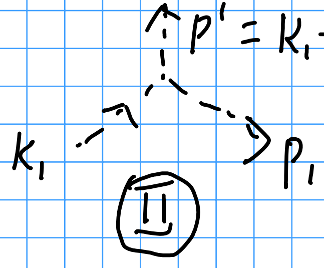
$$A_{fi}^{(I)} = (2\pi)^4 \delta^{(4)}(k_1 + k_2 - p_1 - p_2) \cdot (-iA)^2 \cdot \frac{i}{p^2 - m_\phi^2} \qquad (2.48)$$

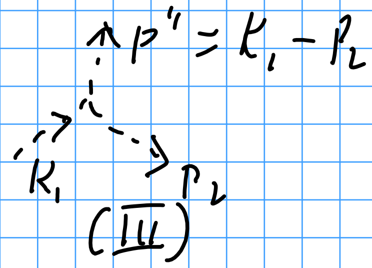
(1st order p. th.)^2: vertex factor

Propagator

<u>Note</u>: Could have started from final state, used initial state as "source" of virtual particle

(2.48) <u>not</u> complete: can attach a <u>final</u> state particle to ϕ_{k_1} and a virtual particle:





Let's compute contribution (II):

$$A_{fi}^{(II)} = -iA \int d^4x \, \underbrace{\phi_{k_1}(x)\, \phi^*_{p_1}(x)}_{\substack{\text{on-shell} \\ \text{(physical)}}} \, \underset{\substack{\uparrow \\ \text{off-shell} \\ \text{(virtual)}}}{\phi^*_{p'}(x)} \qquad (2.49)$$

$\phi_{p'}(x)$ due to "source" of other two fields:

$$(\partial_\mu \partial^\mu + m_\psi^2)\, \phi_{p'} = -A\, \phi_{p_2} \phi^*_{k_2} = -A\, e^{-ix\cdot(p_2 - k_2)}$$

$$\Rightarrow \phi_{p'}(x) = \frac{A}{p'^2 - m_\psi^2} e^{-ix\cdot(p_2 - k_2)}, \quad p' = p_2 - k_2 = k_1 - p_1$$

$$\Rightarrow A_{fi}^{(II)} = \frac{-iA^2}{p'^2 - m_\psi^2} \int d^4x \, e^{-ix\cdot(k_1 + k_2 - p_1 - p_2)}$$

$$= (2\pi)^4 \delta^{(4)}(k_1 + k_2 - p_1 - p_2)\, (-iA)^2 \frac{i}{p'^2 - m_\psi^2} \qquad (2.50)$$

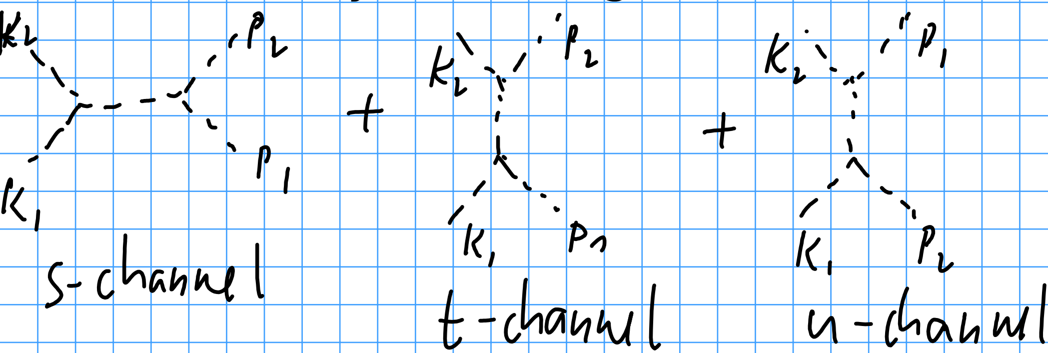
Diagram (III) analogous, with $p' \to p'' = k_1 - p_2 = p_1 - k_2$

$$\Rightarrow A_{fi} = (2\pi)^4 \delta^{(4)}(k_1 + k_2 - p_1 - p_2) \cdot (-iA^2) \cdot \left[\frac{i}{p^2 - m_\psi^2} + \frac{i}{p'^2 - m_\psi^2} + \frac{i}{p''^2 - m_\psi^2}\right]$$

$$A_{fi}^{(\phi^3)} = (2\pi)^4 \delta^{(4)}(p_1 + p_2 - k_1 - k_2) \cdot (-iA)^2 \cdot \left[\frac{i}{s - m_\psi^2} + \frac{i}{t - m_\psi^2} + \frac{i}{u - m_\psi^2}\right] \qquad (2.51)$$

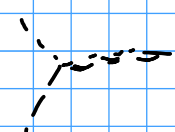
$$s = (p_1 + p_2)^2 = (k_1 + k_2)^2; \quad t = (p_1 - k_1)^2 = (p_2 - k_2)^2; \quad u = (p_1 - k_2)^2 = (p_2 - k_1)^2$$

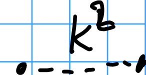
Associated Feynman diagrams for 2 --> 2 scattering:



Note: Needed two powers of coupling A: Can make contact to 2nd order perturbation theory.

Our results can be summarized in the following Feynman rules for calculating the contribution of a given diagram to -i F:

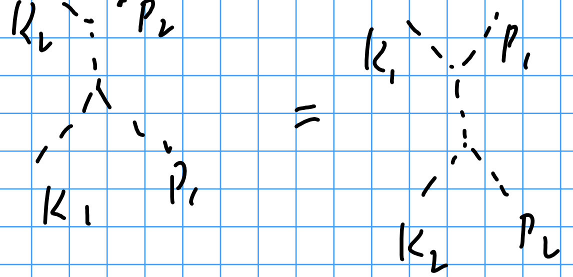
*) Write factor -i A for each vertex  $: -iA \equiv i \dfrac{\partial^3 \mathcal{L}}{\partial \phi^3}$ (2.56)

*) Write a factor $\dfrac{i}{k^2-m_\phi^2}$ for each internal propagator with 4-mom. k flowing through it:  $: i/k^2-m_\phi^2$ (2.57)

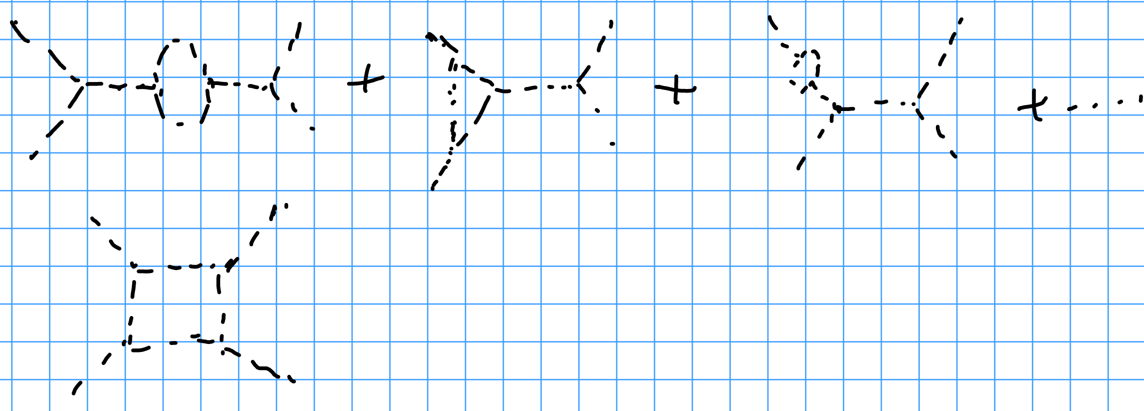
Note: vertex and propagators are independent of each other!
Propagator: from free Lagrangian

Vertex: from interaction

In order to compute the transition amplitude for a given process: have to sum all distinct Feynman diagrams that can be drawn using the given vertices and propagators. Note: diagrams that are related to each other by swapping vertices are not distinct:



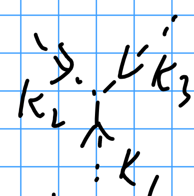
*) Can also have closed loops in diagrams; e.g. 2 --> 2 scattering in ϕ^3 theory:



These can be treated using a slightly modifed set of Feynman rules (for A_{fi}, not for F):

-) For each vertex w/ incoming momenta k_1, k_2, k_3:

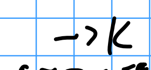
factor $-iA \cdot (2\pi)^4 \delta^{(4)}(k_1 + k_2 + k_3)$



(2.58)

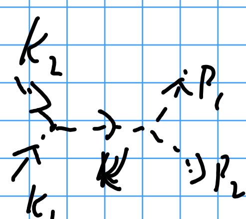
4-mom. conservation enforced at each vertex!

-) For each internal propagator w/ 4-momentum k flowing through it: $\int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2 - m_\phi^2}$



(2.59)

E.g.



$$: A_{fi} = (-iA)^2 \cdot \int \frac{d^4k}{(2\pi)^4} (2\pi)^4 \delta^{(4)}(k_1 + k_2 - k) \cdot (2\pi)^4 \delta^{(4)}(k - p_1 - p_2) \cdot \frac{i}{k^2 - m_\phi^2}$$

$$= (-iA)^2 (2\pi)^4 \cdot \delta^{(4)}(k_1 + k_2 - p_1 - p_2) \cdot \frac{i}{(k_1 + k_2)^2 - m_\phi^2} \quad \checkmark\checkmark$$

-) Need additional symmetry factor if there are closed loops w/ identical particles

The calculation of some 1-loop diagrams will be described in the lecture (QED).