Theoretical Particle Physics 1 (WS 23/24)

Homework No. 10 (Jan. 14, 2024)

To be handed in by Sunday, Jan. 21!

Evaluation of this Class

As usual in Bonn, you have the chance to evaluate this class, including the performance of the professor and your tutor. In order to do so, you should fill out an on–line questionnaire on eCampus: `https://fspha.de/physics615` . If you haven't done this already, you first have to join the corresponding "course" on eCampus created by the student representatives ("Fachschaft"); this you need to do only once in your lifetime, here: `https://fspha.de/fachschaft` .

A high participation rate is in the interest of the department as a whole (since it saves us the trouble of introducing additional evaluation schemes) and of future students taking this class (since constructive criticism is always welcome).

Quickies

Q1: What is Goldstone's theorem?

Q2: What is the "unitary gauge" in a theory with "spontaneously broken" (better: hidden) local $U(1)$ invariance?

Q3: Which vertex involving scalar and gauge bosons *only* exists if the theory is "higgsed" (i.e. if the gauge symmetry is hidden)?

Q4: Give an expression for the vacuum expectation value of the Higgs field, assuming the mass of the gauge boson and the [$U(1)$] gauge coupling are known.

1. $U(1)$ **Higgs mechanism**

   The potential of a complex scalar Higgs field ϕ carrying a $U(1)$ charge is given by

   $$V_{\text{Higgs}} = m^2|\phi|^2 + \frac{\lambda}{2}|\phi|^4 \,. \tag{1}$$

   As discussed in class, this potential has its minimum away from the origin, i.e. for non–vanishing vacuum expectation value $v \equiv \langle\phi\rangle$, if $m^2 < 0$. Now, let's take the vev as

   $$\langle\phi\rangle = v\mathrm{e}^{i\beta} \,, \tag{2}$$

   where v and β are positive constants; the case discussed in class corresponds to $\beta = 0$, but here β should be left arbitrary.

   (a) Obtain the mass matrix $\mathcal{M}^2$ for the real fields η and ξ, defined through the decomposition of the complex field ϕ:

   $$\phi(x) = v\mathrm{e}^{i\beta} + \frac{1}{\sqrt{2}}[\eta(x) + i\xi(x)]. \tag{3}$$

The mass matrix is derived from the second derivatives of the potential (1) at the minimum:

$$\mathcal{M}^2_{ij} = \frac{\partial^2 V(z_k)}{\partial z_i \partial z_j}|_{\phi=\langle\phi\rangle}, \tag{4}$$

where $z_i \in \{\eta, \xi\}$.

(b) What are the eigenvalues and the corresponding eigenstates of this mass matrix? As discussed in class, these are the physical masses and physical (freely propagating) states in a model with global $U(1)$ invariance. *Hint:* In a 2−state problem like this, the orthogonal matrix O introduced in eq. (3.43) in class can be parameterized by a single "mixing angle" γ:

$$O = \begin{pmatrix} \cos\gamma & \sin\gamma \\ -\sin\gamma & \cos\gamma \end{pmatrix}. \tag{5}$$

The value of γ is fixed by the diagonalization condition

$$\mathcal{M}^2 O^T = O^T \mathrm{diag}(m_1^2, m_2^2), \tag{6}$$

where diag(a, b) is the diagonal matrix with a and b on its diagonal. The matrix equation (6) also fixes the eigenvalues m_1^2, m_2^2 of the mass matrix $\mathcal{M}^2$. (All this should be familiar to you e.g. from diagonalizing 2−state Hamiltonians in quantum mechanics.)

(c) Write the potential in terms of η and ξ, by inserting the ansatz (3) into eq.(1), and using the expression $v = \sqrt{-m^2/\lambda}$, see eq.(3.37) in class.

(d) Show that the physical (mass) eigenstates h_1 and h_2 are given by

$$h_i = \sum_k O_{ik} z_k. \tag{7}$$

Invert this expression, i.e. solve for the original states z_1 and z_2, and use this result in order to express the potential V_{Higgs} in terms of h_1 and h_2. Compare your result with eq.(3.39) derived in class.