Theoretical Particle Physics 1 (WS 23/24)

Homework No. 13 (Jan. 21, 2024)

To be handed in by Sunday, Jan. 28!

Evaluation of this Class

As usual in Bonn, you have the chance to evaluate this class, including the performance of the professor and your tutor. In order to do so, you should fill out an on–line questionnaire on eCampus: `https://fspha.de/physics615` . If you haven't done this already, you first have to join the corresponding "course" on eCampus created by the student representatives ("Fachschaft"); this you need to do only once in your lifetime, here: `https://fspha.de/fachschaft` .

A high participation rate is in the interest of the department as a whole (since it saves us the trouble of introducing additional evaluation schemes) and of future students taking this class (since constructive criticism is always welcome).

Quickies

Q1: What is the Lagrangian for *(i)* a free Dirac fermion ψ; *(ii)* a free complex scalar ϕ; *(iii)* a free spin–1 boson A_μ?

Q2: Give the complete gauge covariant derivative acting on a left–handed quark in the Standard Model.

Q3: Which electroweak gauge vertices exist in the SM? *(i)* $W^+W^-Z^0$; *(ii)* $Z^0Z^0Z^0$; *(iii)* $W^+W^-Z^0\gamma$; *(iv)* $Z^0Z^0Z^0Z^0$.

Q4: Which of the following two–body decays can occur in the Standard Model at tree–level, assuming all external particles to be on–shell? *(i)* $W^+ \to e^+\nu_e$; *(ii)* $Z^0 \to \tau^+\tau^-$; *(iii)* $\mu^- \to e^-\gamma$; *(iv)* $Z^0 \to h\gamma$, where h is the single physical Higgs boson.

1. **Gauge invariance in the Abelian Higgs model**

   We now "gauge" the global $U(1)$ symmetry in the scenario analyzed in the second problem of last week, by elevating it to a local symmetry. The kinetic term of the $U(1)$ gauge boson A and that of the Higgs field ϕ are given by

   $$\mathcal{L}_{\text{kin}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D^\mu\phi)^\dagger(D_\mu\phi)\,, \tag{1}$$

   where

   $$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu\,, \quad D_\mu\phi = \partial_\mu\phi + igA_\mu\phi\,, \tag{2}$$

   with g being the $U(1)$ gauge coupling constant. (Without loss of generality the $U(1)$ charge of the Higgs field ϕ has been set to 1.)

(a) What is the mass of the $U(1)$ gauge boson, M_A, induced by the vev $\langle\phi\rangle = ve^{i\beta}$? (Recall that $\langle\phi\rangle$, and hence both v and β, must be constant.)

(b) What is the unitary gauge for this choice of vev? To see what this means for the phase of $\phi(x)$, rewrite the decomposition of Eq.(3) of last week in terms of the eigenstates h_1 and h_2 ($m^2_{h_2} > m^2_{h_1}$) of the mass matrix $\mathcal{M}^2$.

(c) As discussed in class, one of the predictions of the Higgs mechanism is a unique vertex (coupling) involving two gauge bosons and one (massive) physical Higgs boson. Show explicitly that this vertex is independent of β.

2. **Higgs Sector of the Standard Model**

In class we had seen that we need an $SU(2)$ doublet ϕ of Higgs bosons in order to generate masses for the charged fermions. Recall that the doublet is a complex representation, i.e. ϕ has to contain four degrees of freedom. The hyperchrge of ϕ is 1/2. In this exercise we analyze the scalar potential; this will turn out to be quite similar to the $U(1)$ toy model discussed in detail in class.

(a) Write down the most general $SU(2) \times U(1)_Y$ symmetric, renormalizable potential for ϕ. *Hint:* There are just two terms.

(b) What condition(s) do the two parameters have to satisfy so that the potential is *(i)* bounded from below; *(ii)* has a minimum at a non–vanishing field value, i.e. ϕ has a non–vanishing vacuum expectation value (VEV)? In the remainder of this problem, we'll assume that these conditions are satisfied.

(c) Show that there is enough gauge freedom to chose the constant VEV to have the form

$$\langle\phi\rangle = \begin{pmatrix} 0 \\ v \end{pmatrix}, \tag{3}$$

with $v \in \mathbb{R}$.

(d) Now write the Higgs field as

$$\phi(x) = \begin{pmatrix} \varphi^+ \\ v + (h + ia)/\sqrt{2} \end{pmatrix}. \tag{4}$$

Use the relation between electric charge Q, weak isospin T_3 and hyperchrge Y to show that φ^+ is a (complex) field with $Q = +1$ while h and a are (real) neutral fields.

(e) Insert the ansatz (4) into the Higgs potential, and show explicitly that φ^+ and a are massless while h gets a positive squared mass. *Hint:* You'll need the expression relating v to the parameters of the potential.

(f) Finally, show that the result of the previous step agrees with the Goldstone theorem.