Theoretical Particle Physics 1 (WS 23/24)

Homework No. 4 (Oct. 29, 2023)

To be handed in by November 5, 2023!

1. Gordon Identities

In class we had described the photon–proton vertex through five form factors. However, for on–shell protons not all of them are independent. Using the Dirac equation and its hermitian adjoint, prove the following identities used in class to reduce the number of independent form factors to three:

(a)
$$\bar{u}(p_2) i\sigma_{\mu\nu} k^\nu u(k_2) = \bar{u}(p_2) \left(2m_p\gamma_\mu - q_\mu\right) u(k_2)\,,$$
where $\sigma_{\mu\nu} = \frac{i}{2}\left[\gamma_\mu\,,\gamma_\nu\right]$, $k = p_2 - k_2$ and $q = k_2 + p_2$.

(b)
$$\bar{u}(p_2) i\sigma_{\mu\nu} q^\nu u(k_2) = -\bar{u}(p_2) k_\mu u(k_2)\,.$$

(c) How do the corresponding identities for an antiproton current look like, i.e. for the spinor combination that is relevant for electron scattering on an antiproton? Take k_2 to be the momentum of the incoming antiproton and p_2 that of the outgoing antiproton; the definitions of $k = p_2 - k_2$ and $q = k_2 + p_2$ are unchanged.

(d) How do the corresponding identities look like for the case of $p\bar{p}$ annihilation via the exchange of a photon in the $s-$channel? Take k_2 to be the momentum of the incoming proton, and p_2 to be the momentum of the incoming antiproton, and use the same definitions of q and k.

2. Electron–Pion Scattering

Pointlike charged pions can be described by the type of complex scalar field discussed in problem 2 of the previous sheet.

(a) Write down the invariant amplitude for elastic electron–pion scattering,
$$e^-(k_1)\pi^+(k_2) \to e^-(p_1)\pi^+(p_2),$$
by combining the Feynman rules of ordinary (fermionic) QED with the rule for the coupling between the pion and the photon derived last week.

(b) So far we have assumed the pion to be pointlike. As well known, it actually has finite extension. We therefore expect its coupling to photons to be modified, as discussed in class for the case of the proton. Use symmetry arguments to show that the most general vertex factor (the factor appearing in the Feynman rule for the vertex coupling a photon to a pion line) takes the form $e(k_2^\mu + p_2^\mu)F(k^2)$, where k_2 and p_2 are the 4–momenta of the incoming and outgoing charged pion (with charge $+e$), and $k = p_2 - k_2$ is the 4–momentum of the photon flowing into the vertex.

(c) Use this result to compute $d\sigma(e\pi^+ \to e\pi^+)/dk^2$. Express the result in terms of Mandelstam$-s = (k_1 + k_2)^2$ and $k^2 = t$. *Hint:* You may use the result (1.72) derived in class for the lepton tensor, i.e. neglect the electron mass.

Note: While $e^-\pi^+$ scattering cannot be directly realized, the "crossed" process, $e^+e^- \to \pi^+\pi^-$, can. It allows to measure the form factor $F(k)^2$ in the timelike domain (i.e. for $k^2 > 0$), and plays a very important role in the determination of hadronic contributions to the anomalous magnetic moment of the muon, which we will discuss in a few weeks.