Advanced Theoretical Particle Physics (SS 24)

Homework No. 1 (Apr. 12, 2024)

To be handed in by Thursday, April 18, 4 p.m.

1. Fermion masses in the SM

(a) In class the quark mass terms were written as

$$\mathcal{L}_{q-\text{mass}} = -\sum_{q=u,d} \overline{q_{jR}}(\mathcal{M}_q)_{jk} q_{kL} + h.c., \tag{1}$$

for $q = u, d$. Use unitary transformations of the form $q'_{L,R} = U_q^{L,R} q_{L,R}$ such that these mass terms are diagonalized, where $q'_{L,R}, q_{L,R}$ are three-component vectors in generation space and $U_q^{L,R}$ are four (in general different) unitary 3×3 matrices. Which combinations of these matrices appear in the interactions of the physical (mass eigenstate) quarks q'_k with W and Z bosons?

(b) In general, it would seem that one has to write the charged lepton mass term in the SM also as matrix

$$\mathcal{L}_{l-\text{mass}} = -\overline{l^-_{jR}}(\mathcal{M}_l)_{jk} l_{kL} + h.c., \tag{2}$$

where l_j stands for one of the three charged leptons. Show that in the SM one can assume without loss of generality that $\mathcal{M}_l$, and hence the matrix of lepton Yukawa couplings, is diagonal. *Hint:* Diagonalize $\mathcal{M}_l$ as in the previous case for quarks; then show that one can always find a basis for the neutrinos such that the matrices diagonalizing $\mathcal{M}_l$ drop out of all vertices in the Lagrangian.

2. Dirac and Majorana masses

A general four–component Dirac spinor can be written in the chiral representation as

$$\psi = \begin{pmatrix} \xi_L \\ \xi_R \end{pmatrix}, \tag{3}$$

where ξ_L and ξ_R are two in general *different* two–component spinors.

(a) A *Dirac* mass term is of the form

$$\mathcal{L}_{\text{Dirac mass}} = -m\overline{\psi_R}\psi_L + h.c.; \tag{4}$$

the quark and charged lepton mass terms discussed in the first problem are of this form. Write this mass term in terms of the two–component spinors ξ_L and ξ_R.

(b) A *Majorana spinor* has to satisfy the condition

$$\psi^C \equiv C\bar{\psi}^T = \psi, \tag{5}$$

where $C = i\gamma^2\gamma^0$ is the charge conjugation matrix and the superscript T indicates transposition. What is the physical meaning of this condition, and what does it imply for the two–component spinors $\xi_{L,R}$?

(c) Show that $(\psi_L)^C$ is a *right-handed* field. To that end, show that C is unitary; that $C^{-1}\gamma_5 C = \gamma_5^T = \gamma_5$ (the second relation holds in the chiral basis); and hence that $P_R(\psi_L)^C = C[\overline{\psi_L} P_R]^T$ whereas $P_L(\psi_L)^C = 0$, where $P_{L,R} = (1 \mp \gamma_5)/2$ are the chiral projectors. Do not assume that ψ satisfies the Majorana condition (5).

(d) A *Majorana* mass term is written as

$$\mathcal{L}_{\text{Majorana mass}} = -\frac{1}{2} M_L \overline{(\psi_L)^C}\psi_L + h.c.\,. \tag{6}$$

Why do we need the factor 1/2 if M_L is to be the physical (pole) mass?

(e) In general one can also write a term anologuous to that in (6), with $L \to R$ everywhere. In general $M_L \neq M_R$ is permitted, but are these two quantities related if ψ is a Majorana spinor, i.e. if it satisfies the condition (5)?

(f) Can a Majorana mass term be introduced for any fermion in the SM, if we require gauge invariance and restrict ourselves to terms in the Lagrangian that are power–counting renormalizable? And can one write a Majorana mass term for the SM neutrinos involving the (vev of the) Higgs boson if non–renormalizable terms are allowed?