Theoretical Particle Physics 2 (SS 24)

Homework no. 2 (April 18, 2024)

To be handed in by Thursday, April 25, 4 p.m.

1) Chiral Symmetry and its Breaking in the SM

A chiral transformation is defined by

$$\psi(x) \to \exp(i\alpha\gamma_5)\psi(x)\,, \tag{1}$$

where α is a real constant (i.e. we are considering global transformations here). The behavior of various fermionic field operators under a chiral transformation can be quite instructive, as the second problem (below) will show.

1. Show that $\exp(i\alpha\gamma_5) = \cos(\alpha) + i\sin(\alpha)\gamma_5$. *Hint:* Recall that $\gamma_5^2 = 1$.

2. Show that the vector current $\overline{\psi}\gamma_\mu\psi$ is invariant under a chiral transformation, but the scalar current $\overline{\psi}\psi$ is not. This means that all SM gauge interactions respect the chiral symmetry, whereas Yukawa interactions and fermion masses explicitly break it. (The quark condensate in QCD also breaks it, albeit spontaneously.) In the SM the latter are the only (perturbative) sources of chiral symmetry breaking.

3. Show that the tensor current $\overline{\psi}\sigma_{\mu\nu}\psi$ is not invariant under the chiral symmetry, where $\sigma_{\mu\nu} = \frac{i}{4}[\gamma_\mu, \gamma_\nu]$. Hence dipole operators of the form $F^{\mu\nu}\overline{\psi}\sigma_{\mu\nu}\psi$ also violate the chiral symmetry, where $F^{\mu\nu}$ is a field strength tensor.

2) Neutrino Decay in the SM

Phenomenologically we need to give some of the known neutrinos nonvanishing (but small) masses. This means that they may decay. Here we will show that in the SM the lifetimes of neutrinos greatly exceed the age of the Universe.

1. Show that there are no tree–level decays of the heavier neutrinos in the SM. *Hint:* This is related to the absence of FCNC.

2. At one–loop level, radiative decays $\nu_i \to \nu_k + \gamma$ become possible. Draw the two kinds of diagrams that contribute in unitary gauge. (In a renormalizable gauge many more diagrams contribute. Nevertheless the actual calculation may be easier in such a gauge. However, here we only need an order of magnitude estimate, we're not going to actually evaluate any loop diagrams.)

3. These diagrams exist only if there is flavor mixing, i.e. there must be some charged lepton ℓ_α that couples to both ν_i and ν_k. Using eq.(1.1) in class, derive the $\nu_i\ell_\alpha W$ vertex factor.

4. Consider one of the diagrams from the second step. Concentrating on the fermionic line, show that the chiral projectors remove the term proportional to the mass of the internal charged lepton.

5. Using the unitarity of the mixing matrix, show that the amplitude would vanish after summing over all internal leptons, if the latter all had the same mass. (This is just like the GIM cancellation in the quark sector.) Given that the diagrams also involve (at least) one heavy W propagator, argue that the amplitude should be suppressed by at least a factor m_τ^2/M_W^2.

6. Argue from QED gauge invariance that the amplitude should have the form of a dipole operator, $\mathcal{A} \sim F^{\mu\nu}\bar{u}_i\sigma_{\mu\nu}u_k \times A$, where $F^{\mu\nu}$ is the electromagnetic field strength tensor and A contains the rest of the amplitude. Using results from the first problem above, the discussion of this problem so far, and dimensional reasoning, argue that
$$|A| \lesssim \frac{eg^2}{16\pi^2} \cdot \frac{m_{\nu_i}}{M_W^2} \cdot \frac{m_\tau^2}{M_W^2} \,. \tag{2}$$
Here e and g are the QED and $SU(2)$ coupling constants.

7. Show that therefore for $m_{\nu_k} \to 0$,
$$|\mathcal{A}| \lesssim \frac{eg^2}{16\pi^2}\frac{m_{\nu_i}^3 m_\tau^2}{M_W^4} \,. \tag{3}$$
Hint: Argue that the spinors and the field strength tensor in the expression for $\mathcal{A}$ each contribute a factor $\lesssim m_{\nu_i}$.

8. Finally, using the fact that the three SM neutrinos have to have masses (well) below 1 eV, show that the (bound on the) lifetime resulting from the estimate (3) is indeed well above the age of the universe, $\tau_U \sim 10^{10}$ y $\sim 5 \cdot 10^{17}$ s $\sim 1/(10^{-42}$ GeV).

3) Neutrino Decay in the Presence of Sterile Neutrinos

Here we extend the SM by assuming that there is (at least) one "sterile" neutrino, which is a complete gauge singlet, but nevertheless mixes with the three "active" SM neutrinos. This means that the index j of the neutrino mass eigenstates now runs from 1 to (at least) 4.

1. Show that now the heavy neutrinos can decay at tree–level, $\nu_i \to \nu_k\nu_j\bar{\nu}_j$, if $m_{\nu_i} > m_{\nu_k} + 2m_{\nu_j}$, where $j = k$ is possible.

2. Show that the lifetime derived from this decay, $\tau(\nu_i)$, can be estimated as
$$\tau(\nu_i) \gtrsim \tau(\mu^\pm) \cdot \left(\frac{m_{\mu^\pm}}{m_{\nu_i}}\right)^5 \frac{1}{\sin^2\theta_{\text{eff}}} \,, \tag{4}$$
where $\tau(\mu^\pm) \simeq 2.2$ μs is the muon lifetime, and θ_{eff} is an effective mixing angle between the singlet and doublet neutrinos.

3. Using the estimate (4), show that even in this case the lifetime of neutrinos with mass at or below the eV scale exceeds the age of the universe.