# Exercise-sheet 1 (16th-17th of april)

## 1 In class exercise:

### 1.1 Compton-scattering:

A photon with energy $E = 100keV$ is scattered by an angle of 90° at an electron at rest. What is the distribution of the energies after the collision? In which direction the electron moves after the collision? The rest mass of the electron is $m_e c^2 = 511keV$. (Hint: Conservation of the four-momentum).

### 1.2 Fouriertransform:

The fouriertransform (FT) of a function $\phi$:$\mathbb{R}^n \longmapsto \mathbb{C}$ is given by:

$$FT\left\{\phi\right\}(\vec{k}) := \frac{1}{(2\pi)^{\frac{n}{2}}} \int_{-\infty}^{+\infty} \mathrm{d}^n x \phi(\vec{x}) \mathrm{e}^{-i\vec{k}\vec{x}} \tag{1}$$

a) Separate the function $\phi(x)$ in an even ($\phi_e(x)$) and an odd ($\phi_o(x)$) contribution. Calculate the fouriertransform and give the real and imaginary parts as solution.

b) Calculate the following fouriertransforms:

- $f(x-a)$, $a$ real
- $f(ax)$, $a$ real and $a \neq 0$
- $f(-x)$

c) Calculate first the FT of the Gaussfunction:

$$g(x) = \frac{1}{\sqrt{a\sqrt{\pi}}} \mathrm{e}^{-\frac{x^2}{2a^2}} \tag{2}$$

and then the product $\Delta x \Delta k$ (uncertainty relation!) with

$$(\Delta x)^2 = \int_{-\infty}^{+\infty} (x - \langle x \rangle)^2 |g(x)|^2 \mathrm{d}x \tag{3}$$

and

$$\langle x \rangle = \int_{-\infty}^{+\infty} x |g(x)|^2 \mathrm{d}x \tag{4}$$

### 1.3 Delta-Function:

The $\delta$-function is defined by:

$$\int_{-\infty}^{+\infty} \delta(x - x_0) f(x) \mathrm{d}x = f(x_0) \tag{5}$$

Show that:

- $x\delta(x) = 0$
- $\delta(ax) = \frac{1}{|a|}\delta(x)$
- $\delta(x^2 - a^2) = \frac{1}{2|a|}(\delta(x-a) + \delta(x+a))$

# 2 Homework - due date: 22nd of April 2009 (30 Points)

## 2.1 De Broglie wavelength (6 points):

An important length-scale in quantum-physics is the de Broglie wavelength. As you most likely know already, it is given by: $\lambda = h/p$. With $h$ the Planck-constant. Following a few more constants to make your calculation easier: $c = 3 * 10^8 m/s$, $h = 6.62606896 * 10^{-34} Js = 4.13566733 * 10^{-15} eVs$

- How does $\lambda$ depend on the kinetic energy for the classical ($v << c$) and for the relativistic ($v \approx c$) case (2 points)?
- Calculate the de Broglie wavelength for a few cases (4 points):
  - an electron: $m_e c^2 = 511 keV$ with $E = 15 eV$
  - a neutron: $m_n c^2 = 939 MeV$ with $E = 10 MeV$
  - a Homo Sapiens: $m = 80 kg$ with $v = 3 km/h$

## 2.2 Fouriertransform and convolution (12 points):

The convolution of two functions $\phi_1$ and $\phi_2$ is defined by:

$$(\phi_1 * \phi_2)(x) = \int_{-\infty}^{+\infty} \mathrm{d}y \phi_1(x-y)\phi_2(y) \tag{6}$$

a) Show that the FT maps a product into a convolution, so that (4 points):

$$\sqrt{2\pi} FT\left(\phi_1\phi_2\right)(x) \equiv \sqrt{2\pi} \int_{-\infty}^{+\infty} \frac{\mathrm{d}k}{\sqrt{2\pi}} \phi_1(k)\phi_2(k)\mathrm{e}^{ikx} = (\phi_1 * \phi_2)(x) \tag{7}$$

b) Show that (4 points):

$$\left[\frac{\partial}{\partial x_\alpha}(\phi_1 * \phi_2)\right](\vec{x}) = \left[\left(\frac{\partial}{\partial x_\alpha}\phi_1\right) * \phi_2\right](\vec{x}) = \left[\phi_1 * \left(\frac{\partial}{\partial x_\alpha}\phi_2\right)\right](\vec{x}) \tag{8}$$

c) Show that: (4 points):

$$\left[\frac{\partial}{\partial k^j}(FT\phi)\right](\vec{k}) = \mathrm{i}[FT(x^j\phi)](\vec{k}) \tag{9}$$

and

$$\left[FT\left(\frac{\partial}{\partial x^j}\phi\right)\right](\vec{k}) = -\mathrm{i}k^j[FT(\phi)](\vec{k}) \tag{10}$$

## 2.3 Hamilton/Lagrange-Formalism (12 points):

Use Lagrange- (6 points) and Hamilton- (6 points) Formalism to derive the equations of motions for a 2-dimensional harmonic oscillator, use once cartesian and once polar coordinates. Use poisson-brackets to show that the angular momentum is conserved. (Hint: revise classical mechanics)