Exercise-sheet 4 (14.-15. Mai)

1 In class exercise:

1.1 Symmetric potential

1.1. Show that the hamiltonoperator $\widehat{H} = -\frac{\hbar^2}{2m}\partial_x^2 + V(x)$ with a symmetric potential $V(x) = V(-x)$ commutes with the parityoperator $\widehat{P}$ (defined as $\widehat{P}f(x) = f(-x)$).

1.2. Show that the following relation is valid: $\widehat{H} = \frac{1}{4}(1+\widehat{P})\widehat{H}(1+\widehat{P}) + \frac{1}{4}(1-\widehat{P})\widehat{H}(1-\widehat{P})$.

1.3. Assuming $u_1(x)$ and $u_2(x)$ are solutions of the Schrödinger-equation to the same eigenvalue E: $-\frac{\hbar^2}{2m}u_i''(x) + (V(x) - E)u_i(x) = 0$. The Wronski-determinante is defined as: $W(u_1, u_2; x) = u_1(x)u_2'(x) - u_1'(x)u_2(x)$. Show that $\partial_x W(u_1, u_2; x)$ vanishes. From this it follows that W vanishes too. Explain why?

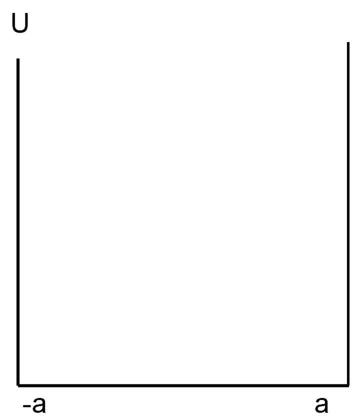
1.4. Use the last property to show $u_1(x) = Cu_2(x)$, with C=const, which means that one-dimensional quantummechanical problems are not degenerated.

1.2 Standing wave

A particle is in a potential well (see picture):

$$V(x) = \begin{cases} U = \infty, & |x| > a \\ 0, & otherwise \end{cases}$$

Calculate the eigenvalue and eigenfunctions. Hint: Consider the boundary- and normalizationconditions.



2 Homework - due date TUESDAY 19th of May 2009 (26 points)

ATTENTION: This time the due date is one day earlier. The reason is that because of the public holiday on thursday, two of the groups will already start on wednesday! For this reason the homework is shorter - however next time you will get a bigger homework again ...

2.1 Harmonic oscillator: coherent states (12 points)

Consider the eigenfunctions ϕ_α of the anhilationoperator a with eigenvalue $\alpha\epsilon\mathbb{C}$.

2.1. Express $\langle\phi_\alpha|\alpha_n\rangle$ through $\langle\phi_\alpha|\phi_0\rangle$ and give an expansion of ϕ_α through the eigenfunctions ϕ_n. Normalize this sum, and derive the time-evolution:
$\phi_\alpha(x,t) = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n e^{-in\omega t}}{\sqrt{n!}} \phi_n(x) e^{-\frac{i\omega t}{2}}$
Why are these coherent states solution of the Schrödinger-equation?

2.2. Calculate $\langle x\rangle$for $\phi_\alpha(x,t)$ and give an interpreation of the result.

2.3. Calculate Δx.

2.2 Particle in a potential (14 points)

A particle is in the following 1-dimensional potential (see picture below):

$$V(x) = \begin{cases} \infty, & |x| > b \\ 0, & b > |x| > a \\ V_0, & a > |x| \end{cases}$$

2.1. Give boundary- and connection-conditions. Derive for $E < V_0$ the equations, so that you can determine the eigenvalues (Hint: Distingish even and odd cases).

2.2. Sketch the solution of those equations.

2.3. Discuss $V_0 \to \infty$.