Exercise-sheet 5 (20th. and 22th. of May)

1 In class exercise:

1.1 Periodic potentials

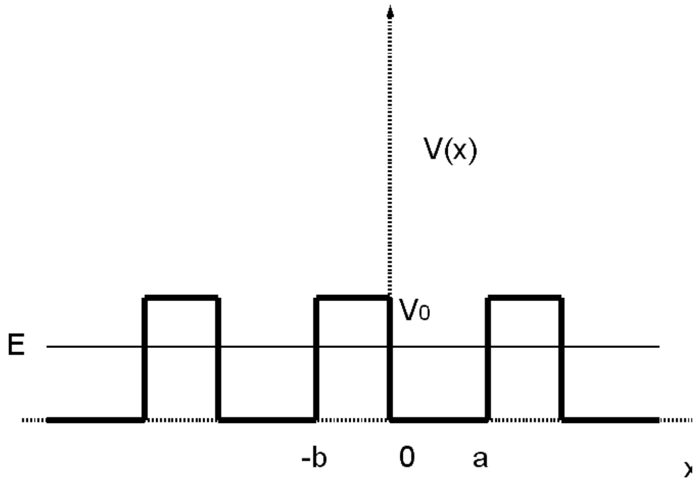
1.1. Show that for a periodic potential $V(x+a) = V(x)$ there are two solutions of the Schrödingerequation for each energy, for which $u(x+a) = \lambda u(x)$ holds.

1.2. Assuming you have two linear independ solutions $u_1(x)$ and $u_2(x)$ of the Schrödingerequation in the interval $0 \leq x \leq a$. How can one obtain the possible energylevels for periodic potentials?

1.3. A periodic potential $V(x)$ of periodicy l=a+b is given (see picture below):

$$V(x) = \begin{cases} 0 & nl \leq x < nl + a \\ V_0 & nl - b < x \leq nl \end{cases}$$

Obtain the energy bands and sketch them for $a = b$ and $\frac{2mV_0a^2}{\hbar^2} = 4$.



2 Homework - due date 27th. of May 2009 (40 points)

2.1 Gaussian wave-packet (12 points)

Consider a gaussian wave-packet in the force-free case, describing a particle.

2.1. At time $t = 0$ the probability of finding the particle shall differ from zero only in a small region around $\overrightarrow{r} = 0$. Further at that time it should move with the momentum $\overrightarrow{p}_0 = \hbar \overrightarrow{k}_0$. Show that then $\Psi(\overrightarrow{r}, 0)$ has the from$\Psi(\overrightarrow{r}, 0) = Ae^{-\frac{r^2}{2a^2} + i\overrightarrow{k}_0 \overrightarrow{r}}$ (Hint: Consider density and flux!).

2.2. Obtain A.

2.3. Consider the special solution of the Schrödingerequation for plane waves (Sheet 3): $\Psi(\overrightarrow{r}, t) = C(\overrightarrow{k}) e^{i(\overrightarrow{k}\overrightarrow{r} - \omega t)}$ with $\omega = \frac{\hbar}{2m} k^2$. The complete solution is then a superposition of all $C(\overrightarrow{k})$. Write down the complete solution of $\Psi(\overrightarrow{r}, 0)$ and use this to calculate $C(\overrightarrow{k})$. Compare the result with 1 and conclude from the result the uncertainty principle (the difference to sheet 3 is, that here with consider the 3-dimensional case!).

2.4. Calculate now $\Psi(\overrightarrow{r}, t)$.

2.5. To gain more insight of the obtained result, calculate density $\rho(\overrightarrow{r}, t)$ and flux $s(\overrightarrow{r}, t) = \frac{\hbar}{2mi}(\Psi^* grad \Psi - \Psi grad \Psi^*)$. Discuss the result (compare with $\rho(\overrightarrow{r}, 0)$).

2.2 Dirac comb (14 points)

Given is a periodic potential formed by a squence of Dirac function with a distance a between them:

$V(x) = \frac{\hbar^2}{m} \Omega \sum_{n=-\infty}^{+\infty} \delta(x + na)$

2.1. Obtain the energy bands for this potential.

2.2. Sketch the first three bands as function of E and ka.

2.3 MOS-FET (14 points)

A particle in a three dimensional space is exposed to a potential which depens on the z-coordinate only:

$$V(z) = \begin{cases} \infty, & z < 0 \\ Fz, & F > 0, z \geqq 0 \end{cases}$$

This is a simplified model for the inversion layer of certain materials such as GaAs found in a MOS-FET.

2.1. Use the ansatz $\psi(r) = \psi_1(x)\psi_2(y)\psi_3(z)$ and show that then the three dimensional Schrödingequation can be reduced to three one-dimensional problems.

2.2. Give the solution for $\psi_1(x)$ and $\psi_2(x)$.

2.3. Which semiclassical quantization condition for the eigenvalues of $\psi_3(z)$ results when using the WKB-approximation?

2.4. Obtain the first three resulting eigenvalues and sketch the wave-function.