Exercise-sheet (28th and 29th of May)

1 In class exercise:

1.1 Angular momentum

1.1. The angular momentum is defined as: $L = x \times p = \frac{\hbar}{i} x \times \nabla$ or written differently: $L_i = \epsilon_{ijk} x_j p_k$. Calculate $[L_i, L_j]$, $[L_i, x_j]$ and $[L_i, p_j]$.

1.2. Calculate L^2 and $[L^2, L_i]$.

1.3. Show that: If an operator commutes with two components of the angular momentum, then the commutator with the third component vanishes too.

1.4. Assuming (L_x, L_y, L_z) is a set of angular momentum operators. Which of the following combinations is one too:
$(-L_x, L_y, L_z)$, $(-L_x, -L_y, L_z)$, $(-L_x, -L_y, -L_z)$.

1.2 Spin

The Pauli-matrices are gives below:
$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
Calculate:

1.1. σ_x^2, σ_y^2, σ_z^2

1.2. $[\sigma_x, \sigma_y]$ and $\{\sigma_x, \sigma_y\}$

1.3. $\sigma_x \sigma_y \sigma_z$, Tr σ_x, Tr σ_y, Tr σ_z, Det σ_x, Det σ_y, Det σ_z

1.4. show that: $\sigma_x \sigma_y = -\sigma_x \sigma_y = i\sigma_z$

2 Homework - due date TUESDAY 9th of June 2009 (36 points)

ATTENTION: The drop-in center for this homework will be rescheduled. Most likely to next wednesday. Please check our webpage concering details.

2.1 Potentialbarrier (14 points)

A particlebeam of energy $0 < E < V_0$ is originating at $-\infty$ and approching a potential barrier:
$V(x) = \begin{cases} V_0 & |x| < a \\ 0 & otherwise \end{cases}$

2.1. Write down the Schrödingerequation and the Ansatz for the wave-function for the three areas. The incoming wave has an amplitude of 1.

2.2. What are the connection conditions of the wavefunction?

2.3. Calculate the transmissioncoefficient T and the reflectioncoefficient R and verify $T + R = 1$

2.4. Show that for the case $E << V_0$ the following is valid:
$T \approx (\frac{4k\mu}{k^2+\mu^2})^2 exp(-2S_0)$ with
$k^2 = \frac{2mE}{\hbar^2}$, $\mu^2 = \frac{2m(V_0-E)}{\hbar}$ and $S_0 = \frac{1}{\hbar}\int_{-a}^{a} dx\sqrt{2m(V(x)-E)}$

2.2 Angular momentum (8 points)

Given are $a^\dagger$ and a a pair of creation and anhilation-operators. For these the usual relations $[a, a^\dagger] = 1$ and $[a, a] = 0$ hold.

2.1. Show that then the operators:
$L^z = \hbar(l - a^\dagger a)$,
$L^+ = \hbar\sqrt{2l - a^\dagger a}a$
$L^- = \hbar a^\dagger\sqrt{2l - a^\dagger a}$
are consistent with the angular-momentum algebra.

2.2. Show: $L^2 = \hbar^2 l(l + 1)$

2.3 Isotropic oscillator in three dimensions (14 points)

In this case the hamiltonoperator is: $H = -\frac{\hbar^2}{2m}\Delta + \frac{1}{2}m\omega^2 r^2$

2.1. Use the Ansatz $\Psi(\overrightarrow{r}) = \Psi(x)\Psi(y)\Psi(y)$ for the stationary solutions to reduce the problem to a one-dimensional one.

2.2. Obtain the degree of degeneracy for a given energy.

2.3. Obtain the wavefunction for the ground-state ϕ_0 and for the first excited state ϕ_i explicitly.

2.4. Show that the angular-momentum operators L_i anhilate the ground-state ϕ_0 .

2.5. Calculate the action of the angular-momentum operators to the wave-functions ϕ_i.

2.6. Show that the ϕ_i are eigenfunctions of L^2 to the same eigenvalue.

2.7. Construct eigenfunctions $\widetilde{\phi_i}$ of L_z by a linear combination of ϕ_i .

2.8. Show that $L^{\pm} = L_x \pm iL_y$ anhilates one of the $\widetilde{\phi_i}$.

2.9. Show that $L^{\mp}$ acting on the $\widetilde{\phi_i}$ which was anhilated by $L^{\pm}$ gives the other $\widetilde{\phi_j}$.