Exercise-sheet 8 (18th and 19th of June)

1 In class exercise:

1.1 Particle with Spin 1

Consider a particle with spin 1.

1.1. Construct the matrix representation of the operators S_x, S_y and S_z in the $|1m\rangle$ basis of S^2, S_z .

1.2. Find the eigenvalues and eigenstates of the $\overrightarrow{n}\,\overrightarrow{S}$ operator, here $\vec{n}$ is the unitvector in arbitrary direction given as: $\vec{n} = cos\phi sin\Theta\vec{e}_x + sin\phi sin\Theta\vec{e}_y + cos\Theta\vec{e}_z$.

2 Homework - due date 24th of June 2009 (30 points)

2.1 Total angular momentum for electron (10 points)

Consider an electron with angular momentum $L = 1$. Calculate the states for the total angular momentum $J = L + S$ - in other words calculate the Clebsch-Gordan-coefficients $\langle m_L m_S | J m_J \rangle$.

2.2 1-d chain of N-Spins (12 points)

The hamiltonian of a 1-d chain of N-Spins (and lattice constant a) with an interaction between nearest neighbors is:
$H = -J \sum_{n=1}^{N} \overrightarrow{S}_n \overrightarrow{S}_{n+1}$, $\overrightarrow{S}_{N+1} = \overrightarrow{S}_1$ and $J > 0$
with $S^{\pm}|m_S\rangle = \hbar\sqrt{S(S+1) - m_S(m_S \pm 1)}|m_S \pm\rangle$

2.1. Show that $S^Z = \sum_{n=1}^{N} S_n^Z$ commutes with H. What does this imply?

The groundstate $|0\rangle$ shall be the state where the spins are directed so that the z-compontents have the maximum $\hbar S$:
$|0\rangle = \prod_{n=1}^{N} |S, m_S = S\rangle_n$ where $|S, m_S\rangle$ are the eigenstates for $\overrightarrow{S}_n^2$ and S_n^Z .

2.2. Let H act on $|0\rangle$. Whats the result for the ground state energy E_0 ?

2.3. Consider the state $|m\rangle = \frac{1}{\hbar\sqrt{2S}} S_m^- |0\rangle$, where Spin m is lowered by one and show:
$H|m\rangle = E_0|m\rangle + J\hbar^2 S(2|m\rangle - |m-1\rangle - |m+1\rangle)$

2.4. The linear combinations $|k\rangle = \sum_{m=1}^{N} e^{ikma}|m\rangle$ of $|m\rangle$ (spin-wave state) are eigenstates of H. Calculate E_k and $S^Z|k\rangle$

Hint: Express H through S^Z and $S^{\pm}$!

2.3 Non-interacting particles (8 points)

N particles, not interacting with each other, move in an external potential $V(\vec{r})$.

2.1. Show that the N-particle Schrödingerequation
$\sum_{i=1}^{N} (\frac{p_i^2}{2m_i} + V(\vec{r}))\Psi(\vec{r}_1, ..., \vec{r}_N) = E\Psi(\vec{r}_1, ..., \vec{r}_N)$
can be reducted to N 1-particle Schrödingerequations.

2.2. Assume the N-particles being identical. Whats the normalized total wave-function for bosons and for fermions?

2.3. What can you say about the ground-state energy for those two cases?