Exercise-sheet 9 (25th and 26th of June)

1 In class exercise:

1.1 Electrons in a cube

Consider N electrons in a cube with dimensions LxLxL, not interacting with each other.

1.1. Show that for the boundary-conditions of the solutions one gets periodic boundary conditions: $\Psi(x, y, z) = \Psi(x + L, y, z) = \Psi(x, y + L, z) = \Psi(x, y, z + L)$.

1.2. Calculate the eigenenergy.

1.3. Calculate the fermi-energy of N electrons.

1.4. Calculate the total energy of the ground state of the N-particle system.

1.2 Bosons und Fermions

Demonstrate which of the following atoms are a Bosons, and which are a Fermions: H, He3, He4.

1.3 Elektrons on a sphere

On a sphere of radius R, there are several electrons (of mass m and spin $\frac{1}{2}$) which do not interact with each other. How many particles can the system have if it is in a state of energy $36\frac{\hbar^2}{2mR^2}$?

2 Homework - due date 1st of July 2009 (30 points)

2.1 Hydrogen-atom (10 points)

Consider the hamiltonian of the H-Atom:
$H = \frac{p^2}{2m} - \frac{e^2}{r}$
Show that the quantummechanical version of the des Runge-Lenz vector:
$\vec{R} = \frac{\hbar}{2m}(\vec{p} \times \vec{L} - \vec{L} \times \vec{p}) - \frac{e^2}{r}\vec{r}$
commutes with the Hamiltonian.
What does this mean for the spectrum of H?

2.2 Perturbation-theory (10 points)

Ein eindimensionales Problem werde durch den folgenden Hamilton-Operator beschrieben:
$H = H_0 + \lambda H_1$, $H_0 = \frac{p_x^2}{2m} + \frac{1}{2}m\omega^2 x^2$, $H_1 = \sqrt{2\hbar m \omega^3}x$

2.1. Calculate the eigenvalues and eigenstates of H.

2.2. Now perform a perturbation-expantion around H_0 and calculate the eigenvalues till λ^2- and the wave-functions till λ-order. Show that the result of the exakt solution and of the perturbation-theory agree.

Hint: Use the represenation with ladder-operators!

2.3 Degenerate-pressure and collaps of a star (10 points)

The pauli-principle leads to a consequence called degenerate pressure, which counteracts the compression of a Fermi-gas. This is given by: $p_{deg} = -\frac{\partial E_{total}}{\partial V}$ where E_{total} is the relation derived in the last part of the first in-class exercise: $E_{total} = \frac{\hbar^2 \pi^3}{10m}(\frac{3n}{\pi})^{\frac{5}{3}} L^3$. After a star burns out - meaning the nuclear reactions stop - this pressure counteracts the collaps (because of gravitation). In this example we assume that other mechanisms can be neglected. Calculate the radius of such a star as a function of the number of nucleons N, by equalizing the pressure caused by gravitation of the nucleons with the degenerate pressure of the electron gas.