Exercise-sheet: special

The following exercises are for those students, who missed the point-criteria slightly. Points you achieve here, will be added to your current result - without change of the 'basis' (the basis is the sum of the points of all regular exercise sheets - this is considered as 100 %). Of course other students are free to do this exercises too, however we will only mark the exercises of students which need the points to fullfill the criteria.

1 Part 1: due at the last tutorial (12 points)

1.1 Quantummechanical rotor in electric field (12 points)

The hamiltonian of a static rotor with moment of inertia I is $H_0 = \frac{1}{2I}\vec{L}^2$. Hint: $Y_{10}(\vartheta,\phi) = \sqrt{\frac{3}{4\pi}} cos\vartheta$, $Y_{20}(\vartheta,\phi) = \sqrt{\frac{5}{4\pi}}(\frac{3}{2}cos^2\vartheta - \frac{1}{2})$.

1.1. What are the eigenvalues and eigenfunctions of H_0 ?

1.2. The rotor be charged and is exposed to an electric field. This leads to an additional potential of $H' = \frac{az}{r}$. Show, which matrix-elements of H' do not vanish. Which order in the perturbation-theory corrects the energies?

1.3. The rotor is now exposed to another electric field, this leads to an additional potential of $H' = b\frac{2z^2-x^2-y^2}{r^2}$. To simplify matters, the perturbation-theory shall be limited to the two lowest energy-levels. Show which matrix-elements (of H') between those eigenstates for those energy-levels, do not vanish.

1.4. Obtain from (3) the relative energy-splittung because of H', which one expects from perturbation-theory of first order. How does the wave-function change?

2 Part 2: due Tuesday 14. Juli 2009 submission directly to your tutor (24 points)

2.1 Pauli-matrices (12 points)

2.1. Show:
$(\vec{\sigma}\vec{A})(\vec{\sigma}\vec{B}) = \vec{A}\vec{B} + i\vec{\sigma}(\vec{A}\times\vec{B})$
were $\vec{A}$ and $\vec{B}$ are operators which commute with $\vec{\sigma}$.

2.2. Show:
$(\vec{\sigma}_1\vec{\sigma}_2)^2 = 3 - 2(\vec{\sigma}_1\vec{\sigma}_2)$

2.3. Use 2. to show that
$\prod_0 = \frac{1}{4}(1 - \vec{\sigma}_1 \vec{\sigma_2})$ and $\prod_1 = \frac{1}{4}(3 + \vec{\sigma}_1 \vec{\sigma}_2)$
are two orthogonal projection-operators. On which of the two-particle-states $|00\rangle, |1{-}1\rangle, |10\rangle, |11\rangle$ the operators $\prod_0$ and $\prod_1$ project to?
Hint: Express $\prod_0$ and $\prod_1$ through $\vec{S}^2 = \frac{1}{4}(\vec{\sigma}_1 + \vec{\sigma}_2)^2$!

2.4. Show that
$\prod_{l+1/2} = \frac{l+1+\vec{L}\vec{\sigma}}{2l+1}$ and $\prod_{l-1/2} = \frac{l-\vec{L}\vec{\sigma}}{2l+1}$
are the analog projection-operators for $j_1 = l$ and $j_2 = \frac{1}{2}$.

2.2 Spin-orbit coupling (12 points)

The hamiltonian of the H-atom, takes the following form when taking into account the relativistic effect of spin-orbit coupling:
$H = \frac{p^2}{2m} + V(r) + V_{L,S}(r)\vec{L}\vec{S}$

2.1. Show that H is invariant of rotation! Is $[H, \vec{L}] = [H, \vec{S}] = 0$ valid too?

2.2. Is H mirrorinvariant?

2.3. Whats the maximum set of commuting observables for H?

2.4. Show that the Schrödingerequation takes the form
$(\frac{p_r^2}{2m} + \frac{\hbar^2 L(L+1)}{r^2} + V_{J,L})\psi = E\psi$
with $V_{J,L}(r) = V(r) + V_{L,S}(r)\Lambda(J,L)$ were $\Lambda(J,L) = \begin{cases} L/2 \\ -(L+1)/2 \end{cases}$